DESIGN OF NEW MATERIALS AND STRUCTURES TO MAXIMIZE STRENGTH AT PROBABILITY TAIL: A NEGLECTED CHALLENGE FOR QUASIBRITTLE AND BIOMIMETIC MATERIALS

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Alfred M. Freudenthal
1906 – 1977

Founder of structural safety. His work epitomized fusion of mechanics and probability.

After him: 50-year SCHISM:

- advanced probability with simplistic mechanics

or

- advanced mechanics with simplistic probability

Make them fuse again!

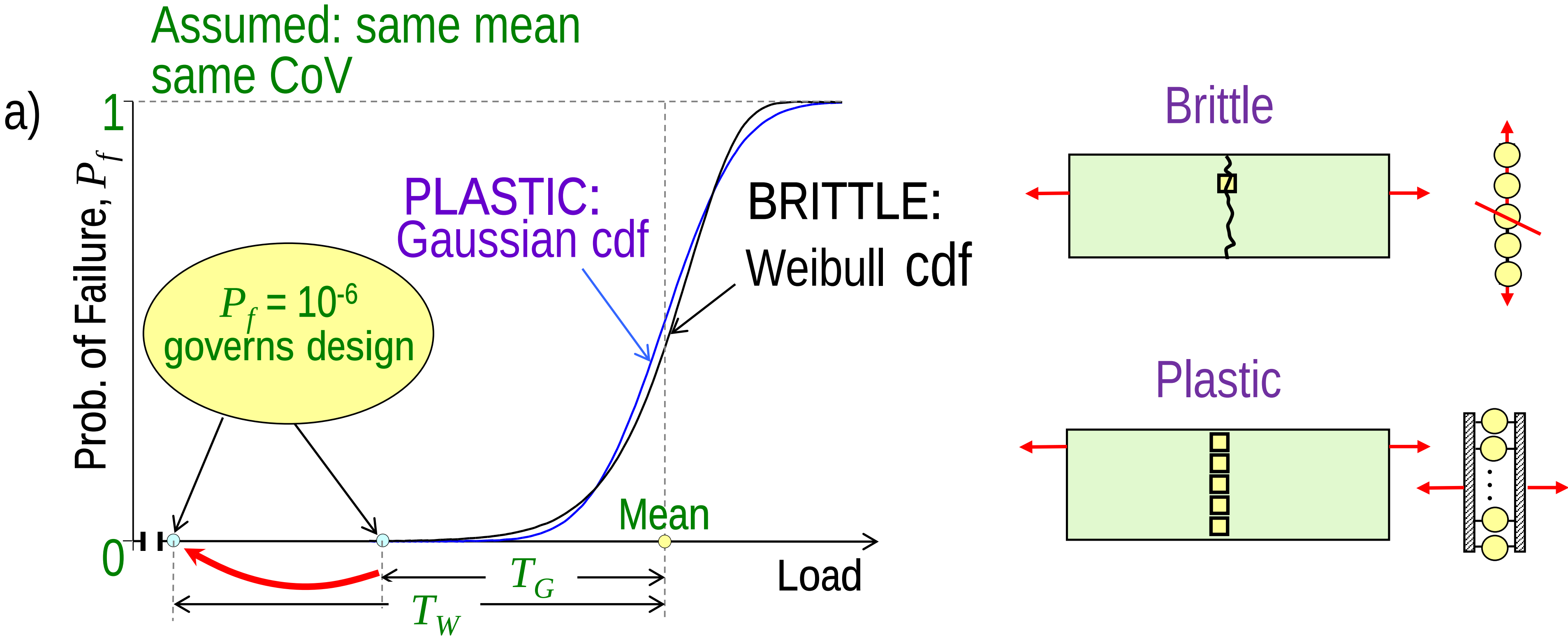
Overlooked:

RELIABLITY-BASED DESIGN OF MATERIALS, NOT JUST STRUCTURES, AND FOCUSED ON THE TAIL

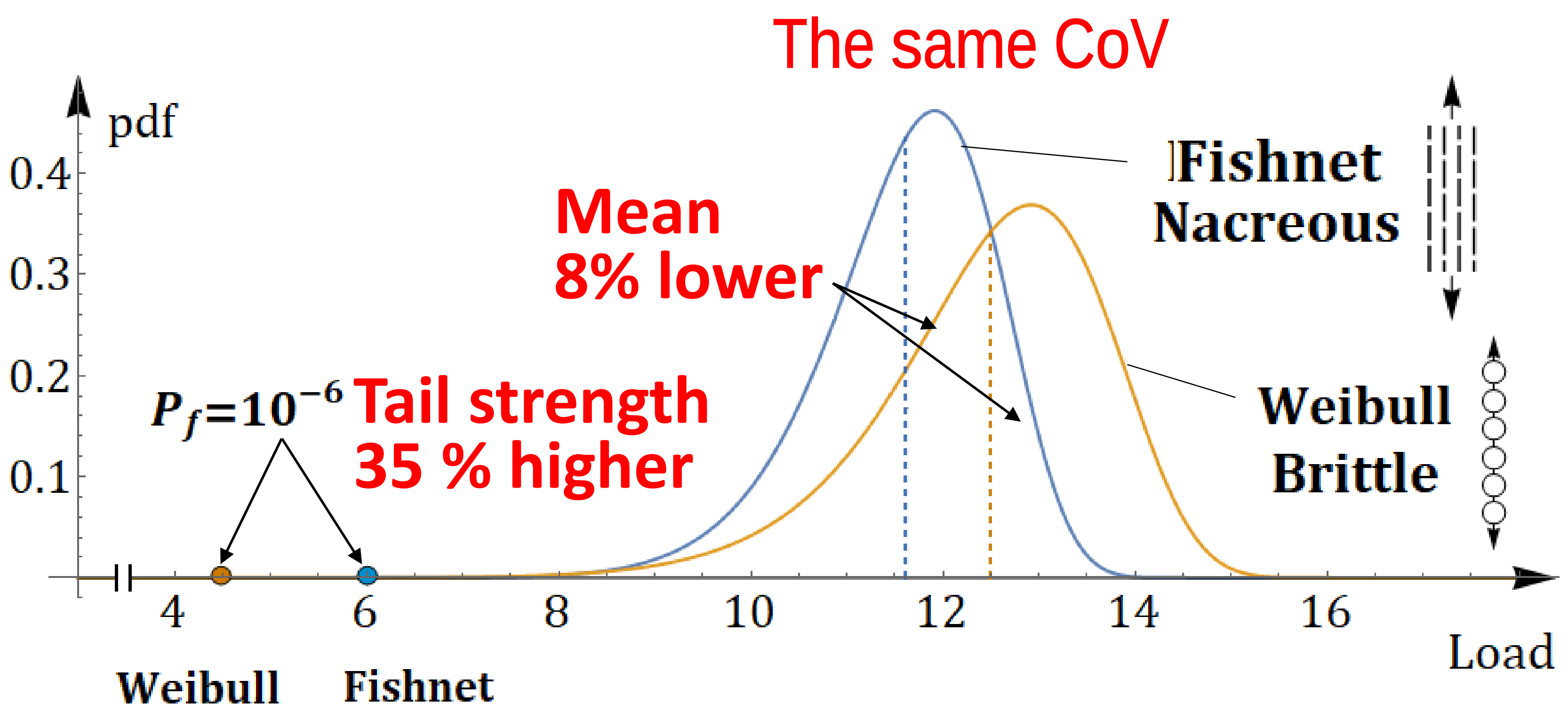
NEEDED: TAIL-RISK DESIGN

- Optimize not the mean material strength but the strength at the tail of 10^{-6} **failure probability**, P_f
- **10^{-6} is the maximum tolerable** P_f for engineering structures
- Controlling material architecture can profoundly alter the strength probability distribution

Example: Huge tail difference between Gaussian (normal) and Weibull cumulative distribution functions (cdf)



In quasibrittle materials, for the same CoV, <u>superior</u> mean strength can lead to <u>inferior</u> strength at the 10^{-6} tail

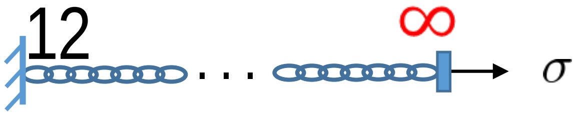


The probability distribution must be known analytically!

Analytically Tractable Strength Models for Failure Probability (incl. Tail)

EXISTING

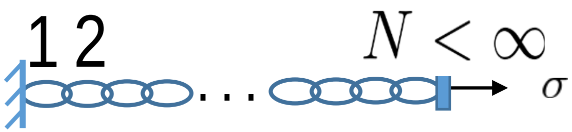
① a)



Infinite weakest-link model

Weibull (1939) distribution; Fisher (1928)

b)



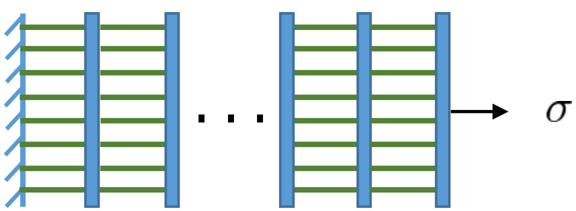
Finite weakest-link model (NU 2005)

② a)

Fiber bundle model (Daniels 1945)

Gaussian distribution

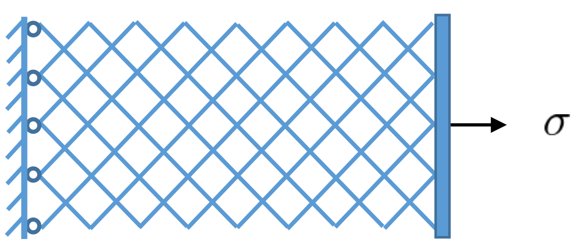
b)



Chain-of-bundles model (Harlow & Phoenix 1985)

NEW

③

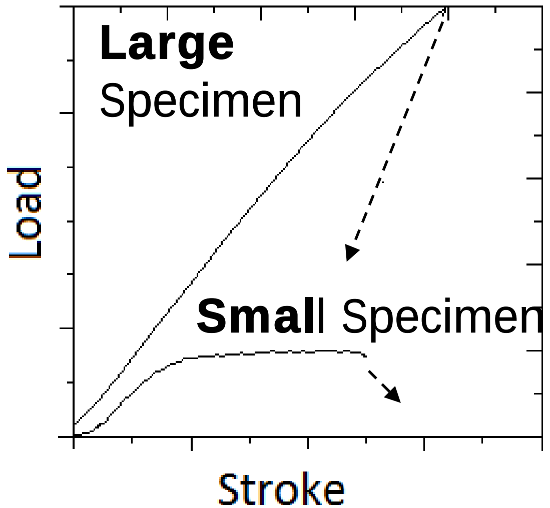


Fishnet statistics (NU 2017)

Quasibrittle Materials

— brittle constituents, but inhomogenity size and the RVE are not << structure size *D*.

Concrete *(archetypical),* ***fiber composites, tough ceramics, rocks, bones, sea ice, rigid foams, dental cements, dentine, nacre, biological shells, cartilage, wood, consolidated snow, particle board, paper, carton, cast iron, thin films, carbon nanotubes, fiber-reinforced concrete, cold asphalt concrete, mortars, masonry, stiff clay, silt, cemented sand, grouted soil, refractories, coal, oil and gas shales, plus all brittle materials on micro- and nano-scales.***



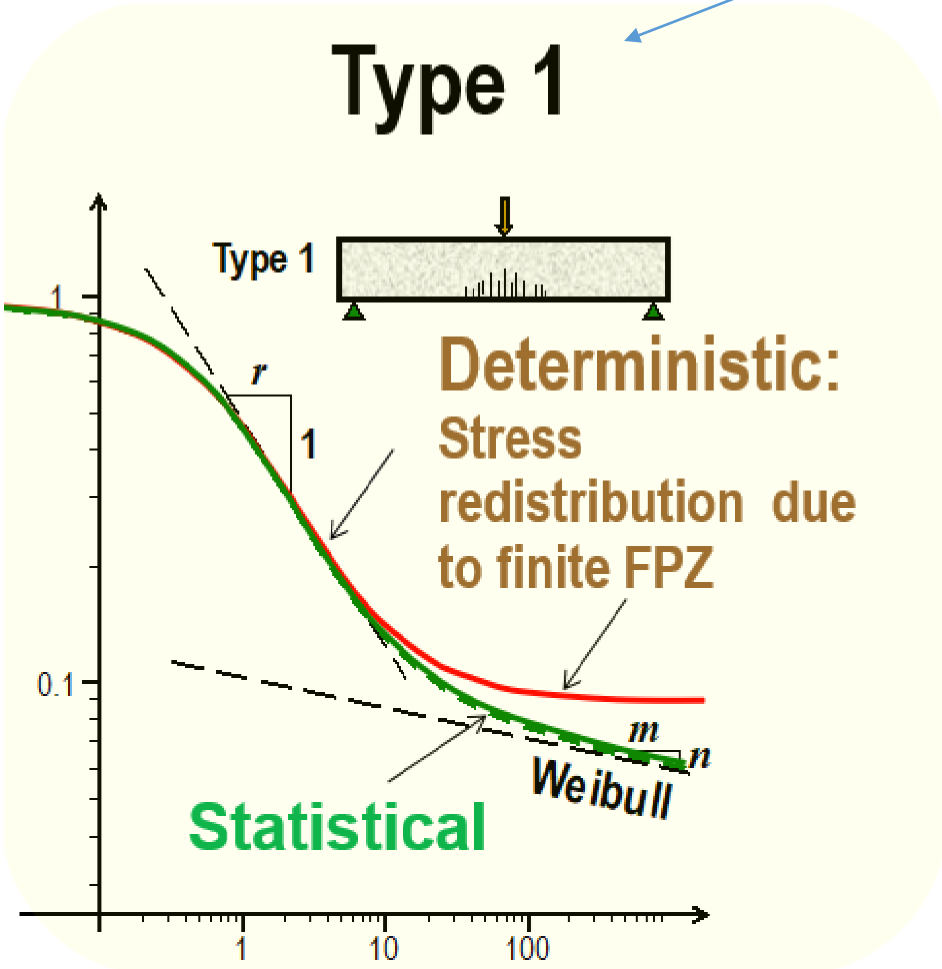
They all exhibit non-negligible material characteristic length.

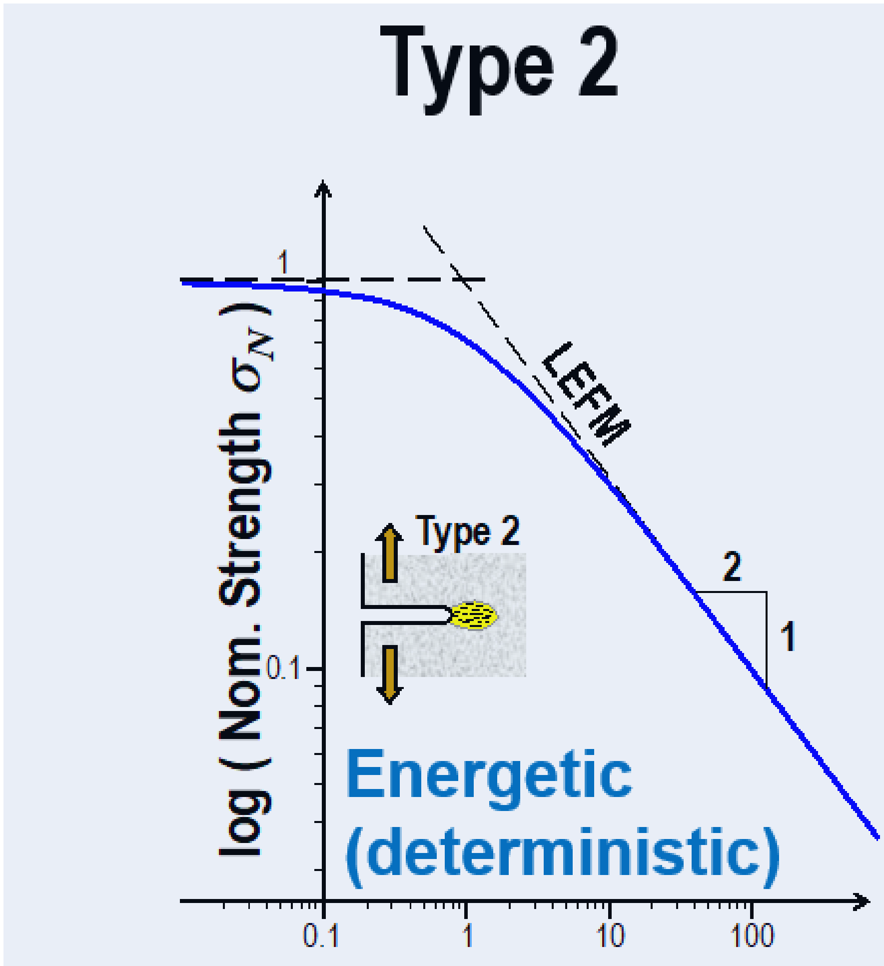
At increasing size *D*, they all transition from ductile to brittle.

I.

Review of Recent Results on Tail Strength Probability of Quasibrittle Randomly Heterogenous Materials

We focus on quasibrittle failures of Type 1

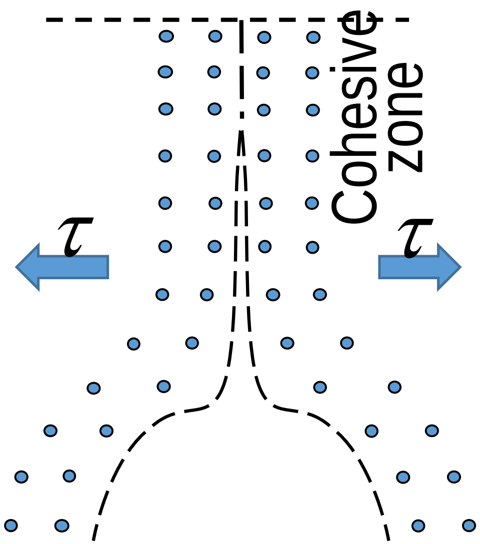


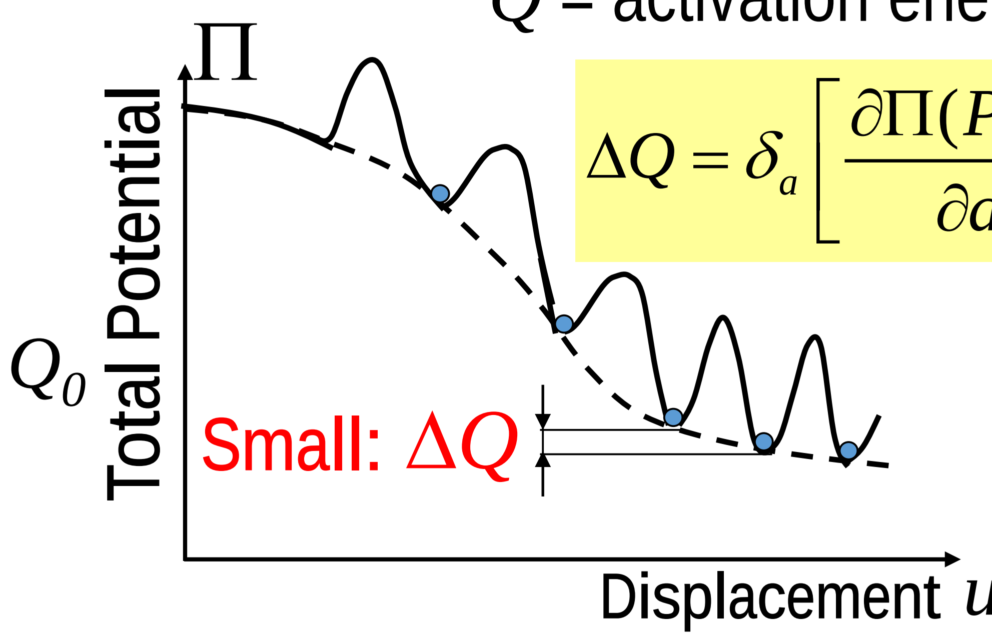


log (Size D)

The only way to determine P_f is on atomistic scale:

Frequency ⇒ Probability





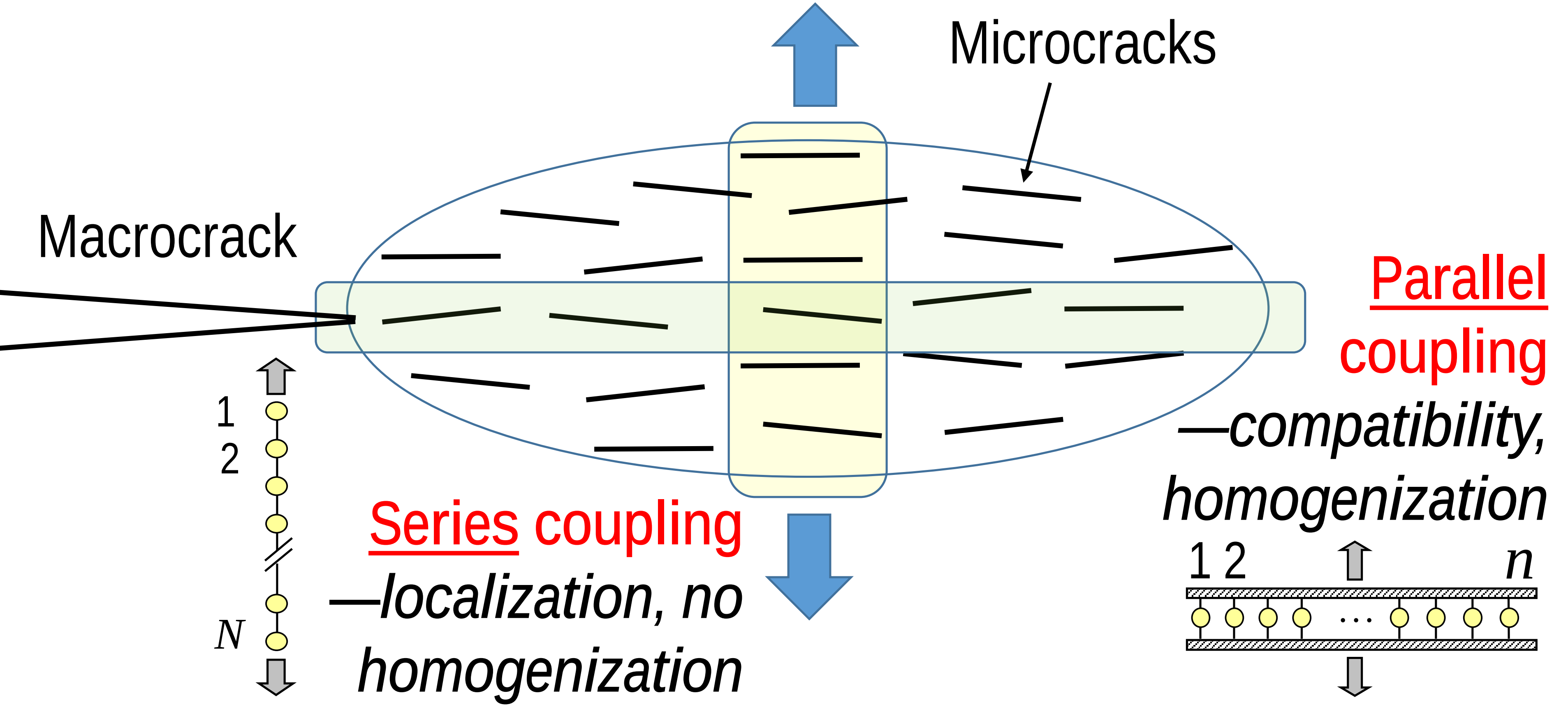
$$\Delta Q = \delta_a \left[\frac{\partial \Pi(P,a)}{\partial a} \right]_P$$

From Kramers' rule of transition rate theory:

$$f_b \sim v_a \left(e^{-(Q_0 - \Delta Q/2)/kT} - e^{-(Q_0 + \Delta Q/2)/kT} \right) \approx 2 v_a e^{-Q_0/kT} \frac{V_a}{2E_a kT} \tau^2$$

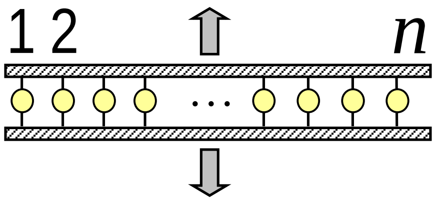
ZP Bažant, J-L Le, MZ Bazant (2009), Proc.,National Academy of Sciences 106, 11484--11489.

How to upscale from nano to macro?
— scale transitions are governed by microcrack interactions in fracture process zone (FPZ)

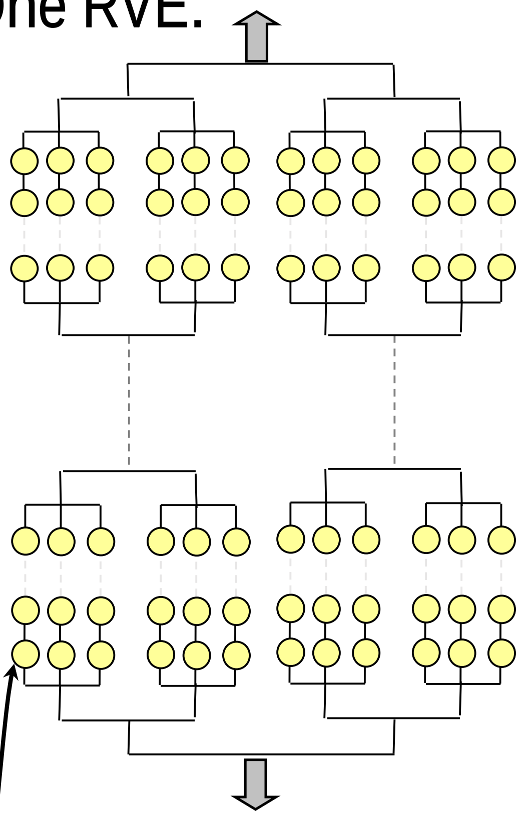


- Power law tail, exponent n = 2 at nanoscale. In scale transitions to macro RVE, power law tail is indestructible, n is increased to 20—50. Parallel couplings increase n, series couplings deepen tail.

Nano-Macro Transition of Tail of Strength cdf

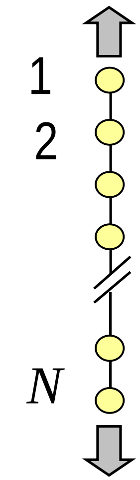


One RVE:

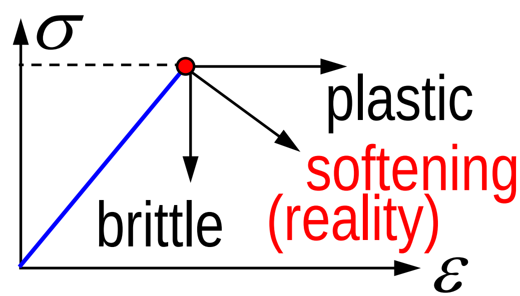


Nanoscale:
cdf tail $\sim\sigma^2$

- In parallel couplings, the **tail exponents are additive.**
- Parallel coupling **shortens the tail** reach by order of magnitude.
- In series couplings, **exponent remains**
- Series coupling **extends the tail reach.**
- Parallel coupling produces cdf with Gaussian core.



- ***<u>Power-law tail</u> with <u>zero</u> threshold is <u>indestructible!</u>***



ZP Bažant, SD Pang (2007). J. of the Mechanics and Physics of Solids 55, 91-134.

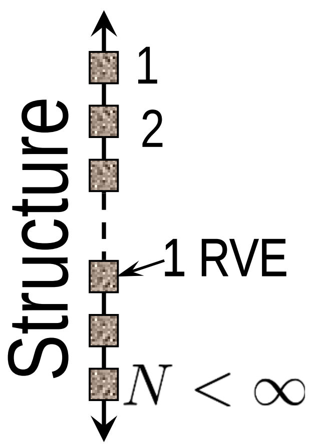
Size Effect on Strength cdf in Weibull Scale

$$P_f = 1 - \left(1 - P_{RVE}\right)^{N_{eq}}$$

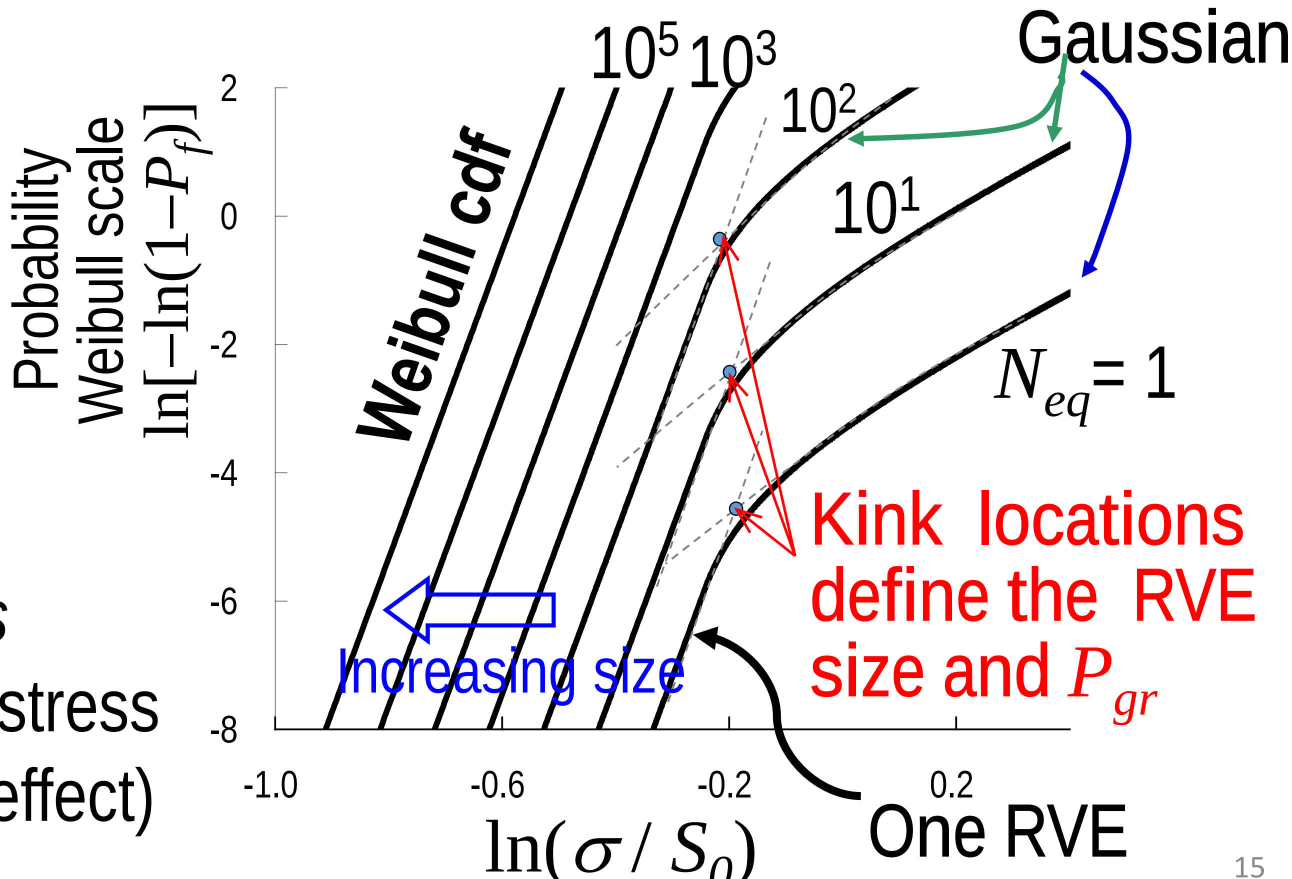
FINITE CHAIN

$$\xrightarrow{N_{eq} \to \infty} \quad P_f = 1 - e^{-(\sigma/s_0)^m}$$

Infinite chain – Weibull
not if quasibrittle



N_{eq} = equivalent N, as modified by the stress field (geometry effect)



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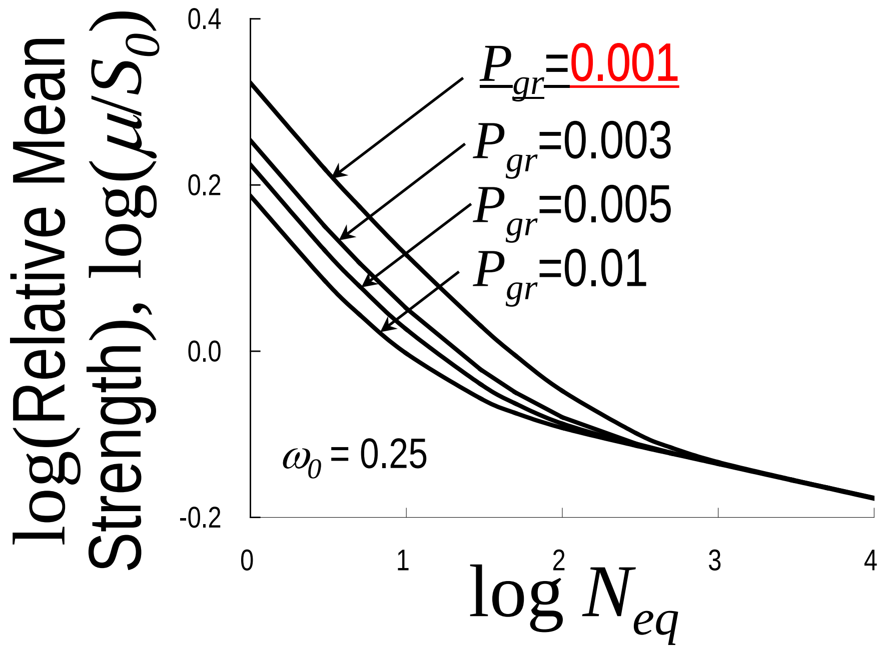
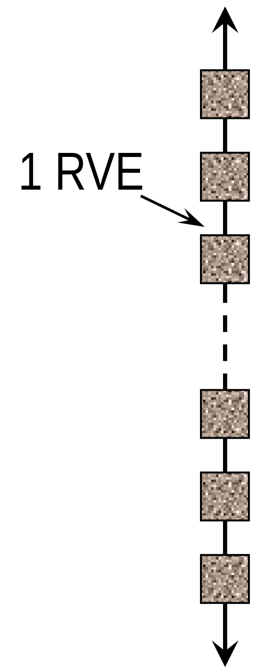
Calibration of P_{gr} by Size Effect





Note: Similar curves are predicted by deterministic nonlocal model.

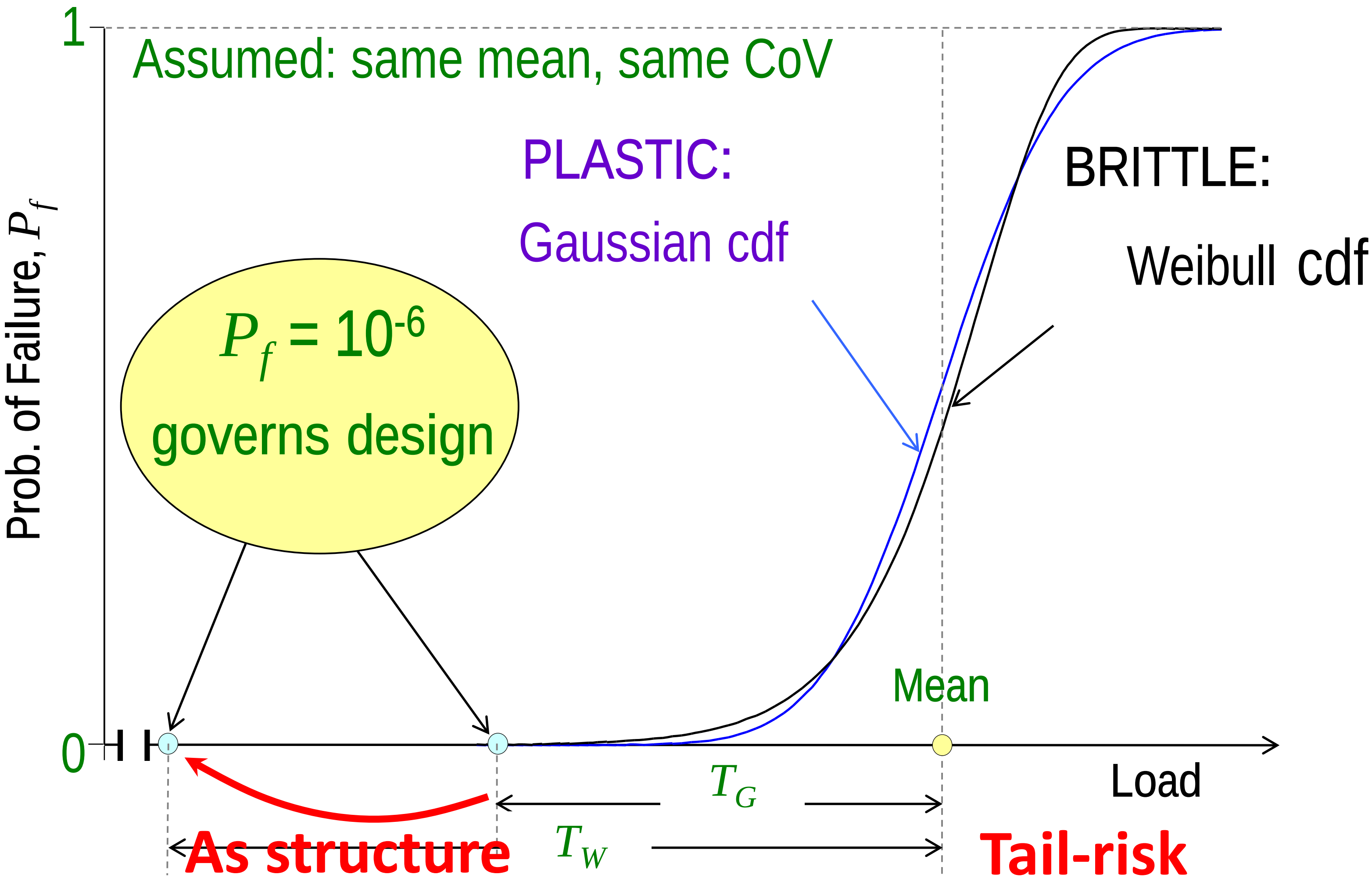
Calibration Result:

$$P_{gr} \approx 0.001$$

Note: Zero threshold!

Tail: $P_f \sim (\sigma - \sigma_u)^n$ $\sigma_u = 0\,!$

ZP Bažant, SD Pang (2007). J. of the Mech. & Physics of Solids 55, 91-134.

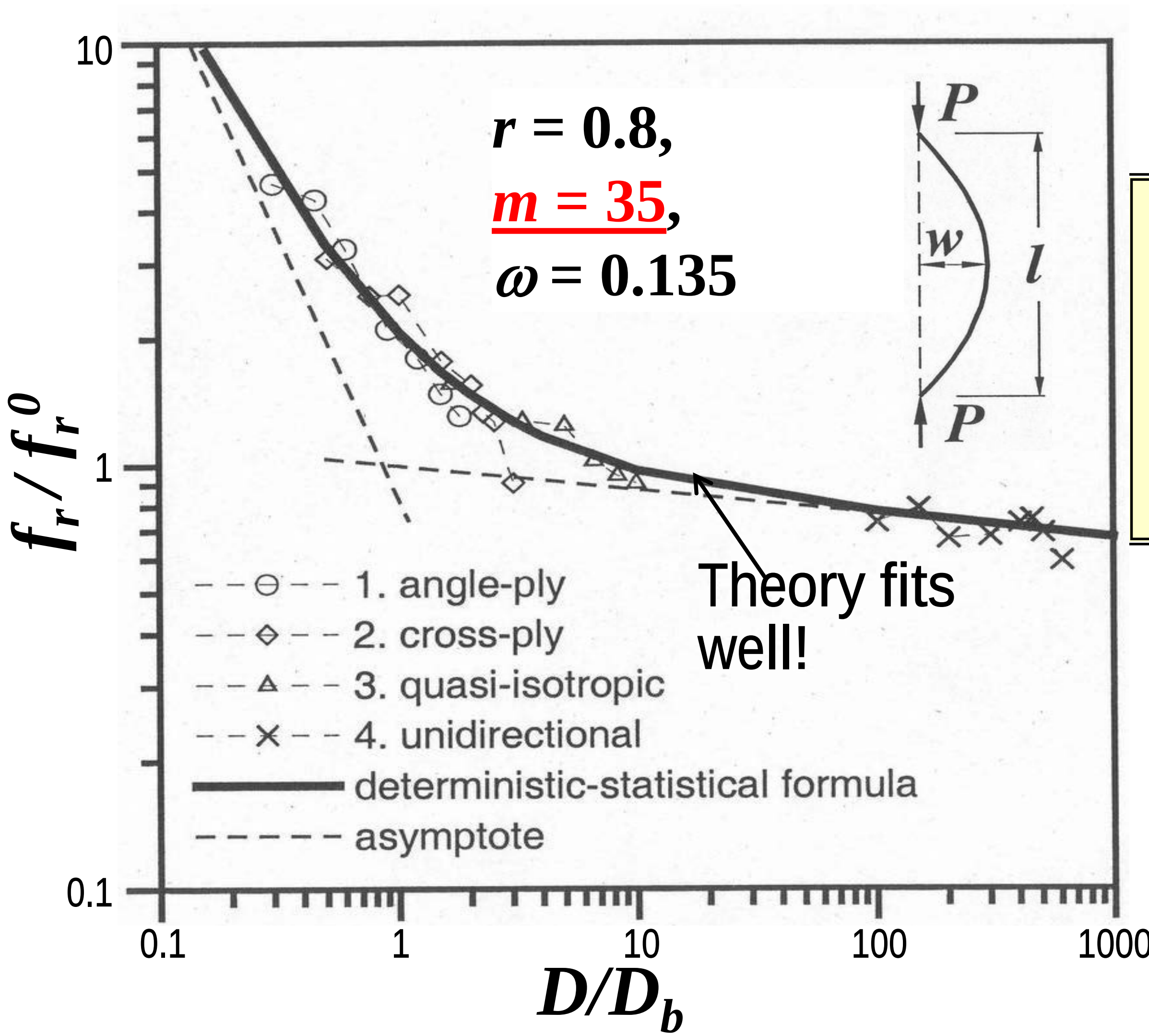


As structure size increases, the tail risk point moves!

Tail-risk design must be <u>size dependent</u>

Size Effect on Flexural Strength of Laminates

Reinterpretation of Jackson's (NASA) Tests



Energetic-Statistical Size Effect Law:

Nominal Strength:

$$\sigma_N = f_r^0 \left(\vartheta^{n_d r/m} + \vartheta \right)^{1/r},$$

$$\vartheta = \frac{rD_b}{D + rsD_b}$$

f_r^0, r, n_d, m, s, D_b = constants,

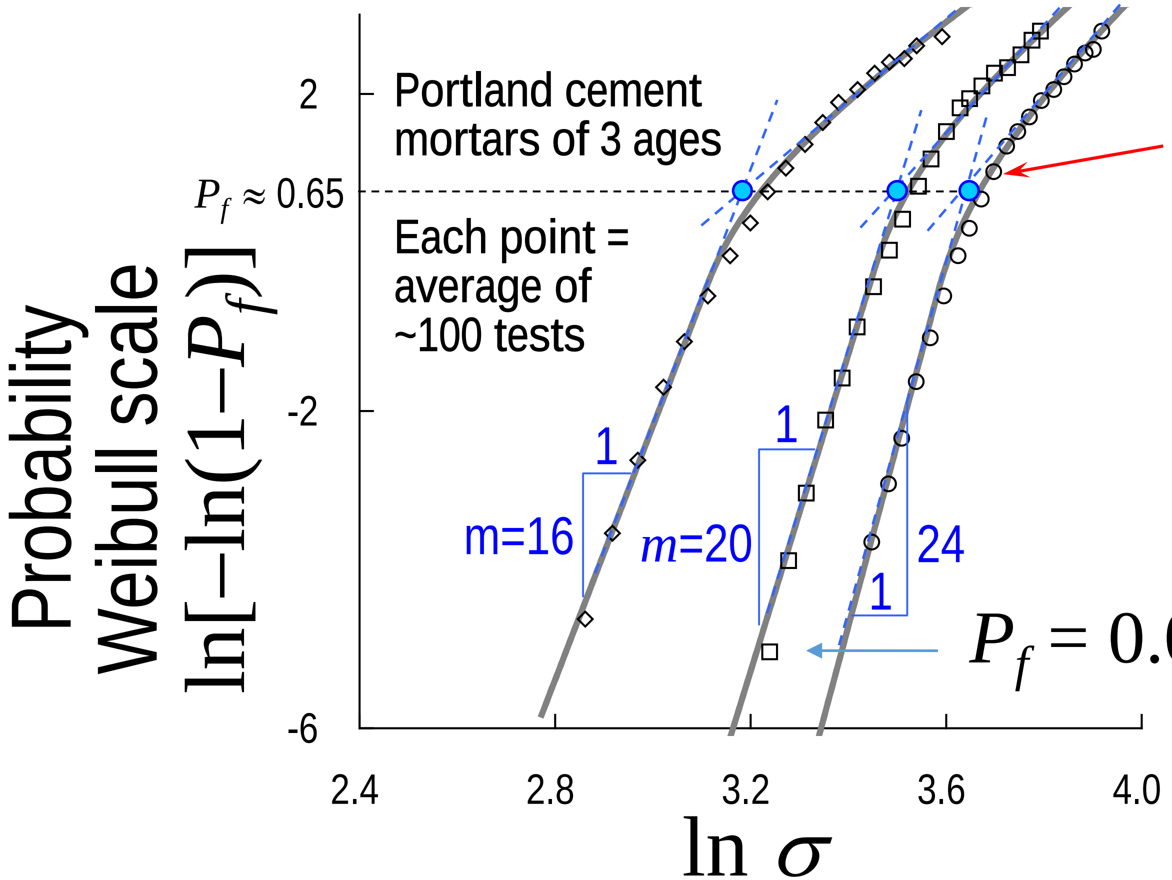
D = char. size of structure,

n_d = no. of dimension for scaling

m = **Weibull modulus**

= material constant, indepent of yarn layout

VALIDATION AND CALIBRATION: Optimal Fit of Weibull's (1939) Monumental Experiments



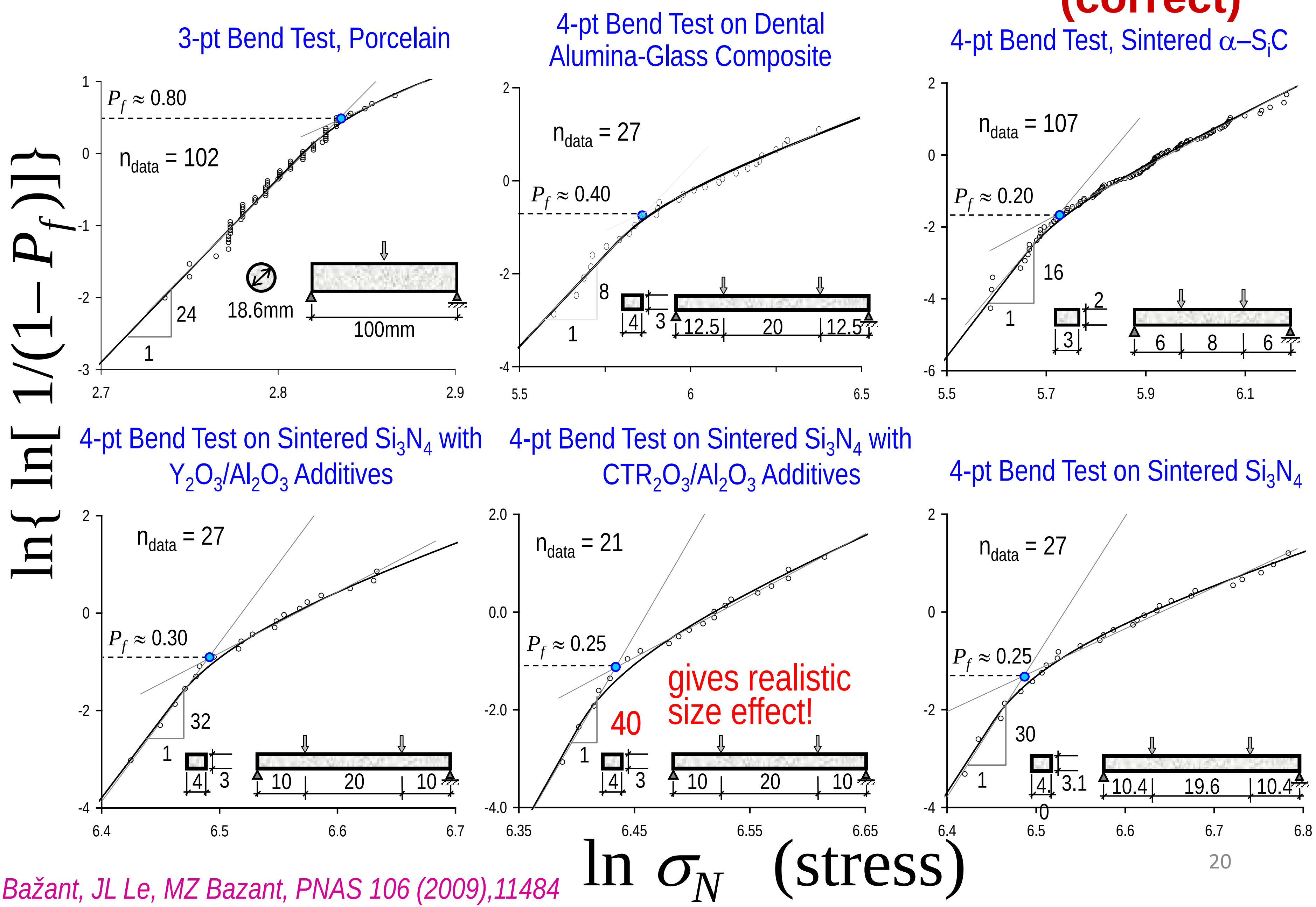
KINK — classical Weibull theory can't explain (finite threshhold is an incorrect remedy)

RVE size ~ 0.6-1.0 cm; Specimen vol. ~ 100-3000 cm^3

$\Rightarrow$ $P_{gr} \approx$ **0.001**

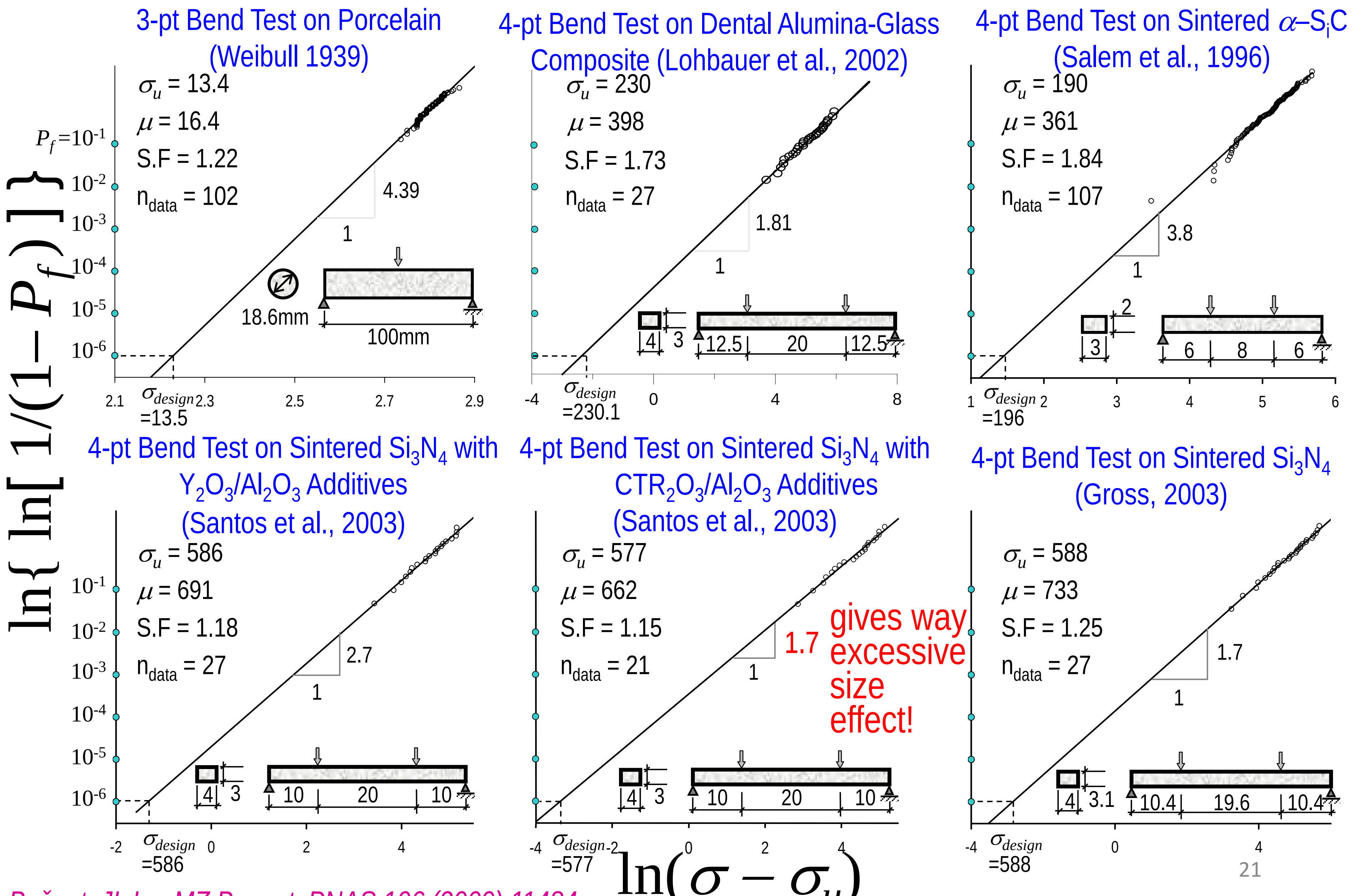
ZP Bažant, SD Pang (2007). J. of the Mechanics and Physics of Solids 55, 91-134.

Optimum Fit by Chain–of–RVEs, Zero Threshold (correct)



Bažant, JL Le, MZ Bazant, PNAS 106 (2009),11484

Optimum Fit by Weibull Theory with Finite Threshold —*incorrect !*



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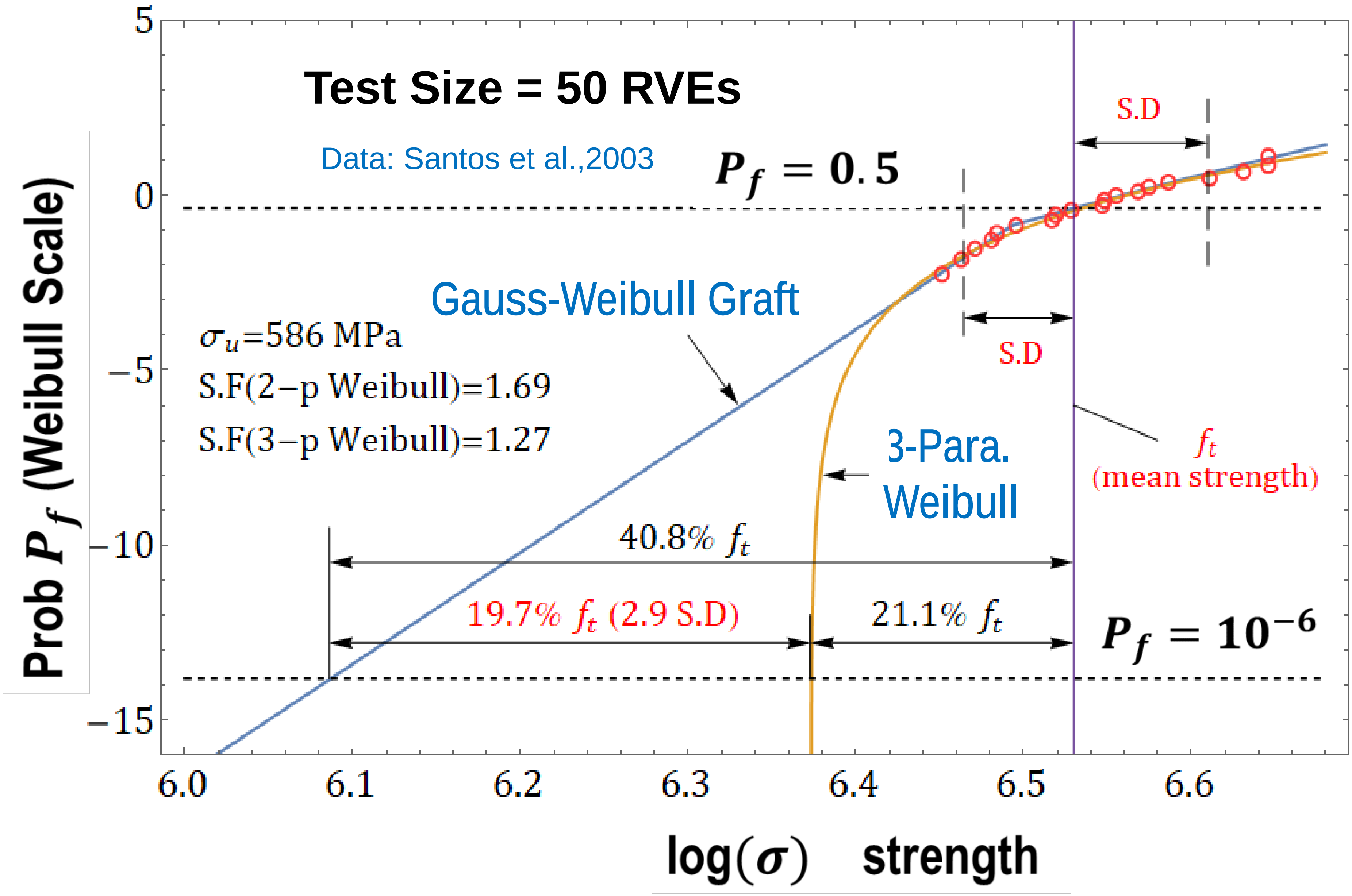
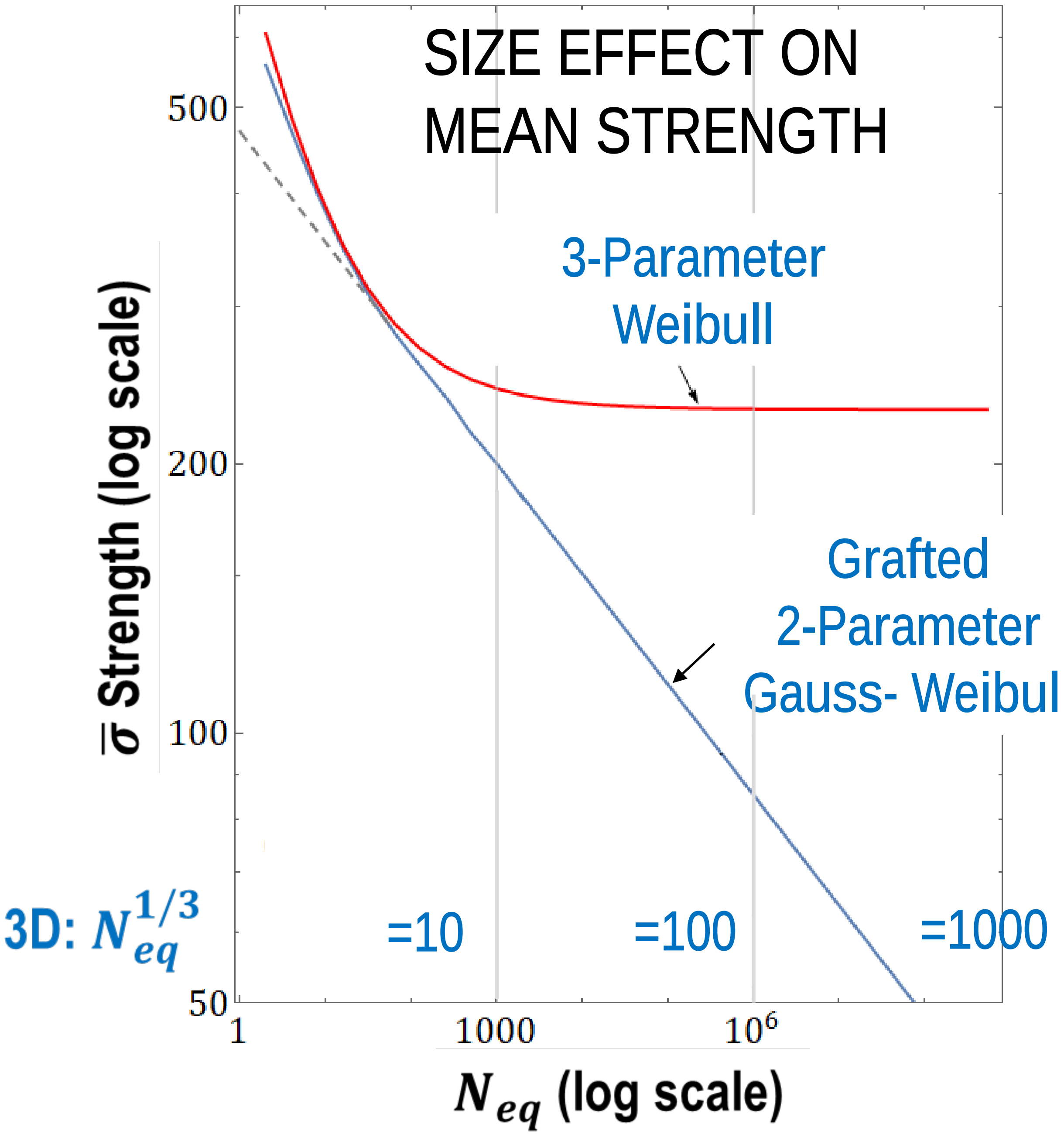


Wrong tail probability is experimentally provable only by size effect



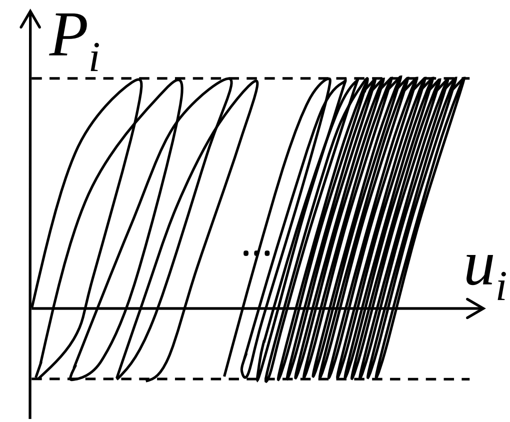
SIZE EFFECT ON
MEAN STRENGTH
3-Parameter
Weibull
Grafted
2-Parameter
Gauss- Weibull
500
200
100
50
$\overline{\sigma}$ Strength (log scale)
3D: $N_{eq}^{1/3}$
=10
=100
=1000
1
1000
10^6
N_{eq} (log scale)

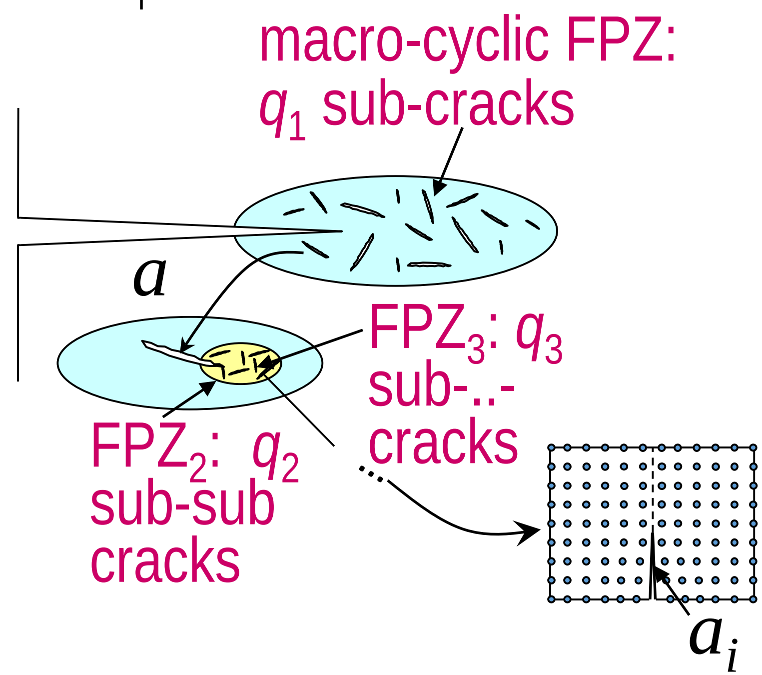
Generalization to Cyclic (or Static) Fatigue Lifetimes

- Atomistic crack-length jumps lead to <u>Paris Law (or Evans Law)</u> at nanoscale, with exponent of stress = 2

- Analytical way of multiscale transition:

Energy dissipation rates in macroscale FPZ and in all atomic scale cracks must be equal





Bažant & JL Le (2009). Engrg. Failure Analysis 16, p. 2521
M. Salviato, K Kirane, & ZP Bažant (2014) JMPS, 440-454