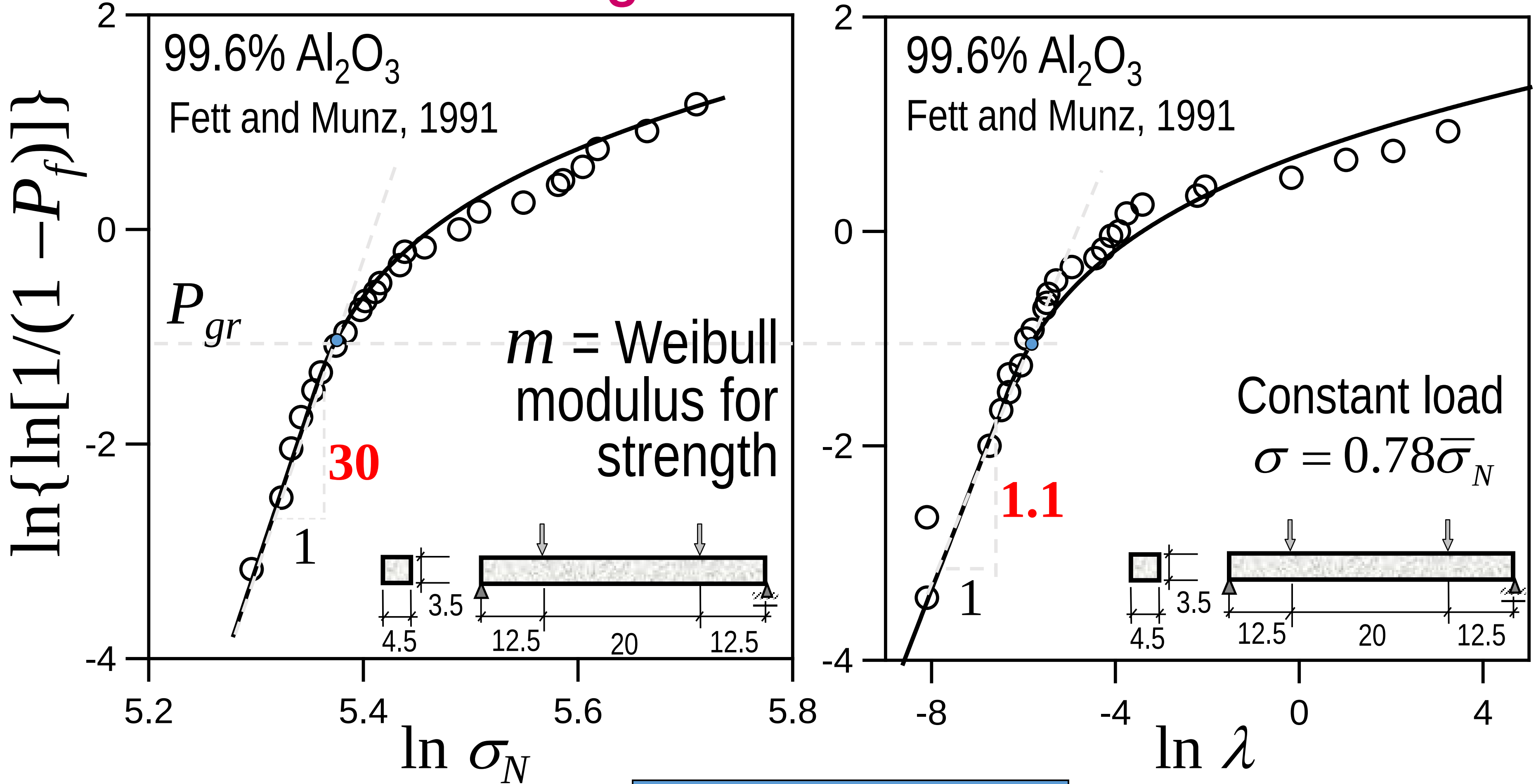
Histograms of Strength and Static Fatigue Lifetime

Calibration: Strength cdf

Prediction: Lifetime cdf

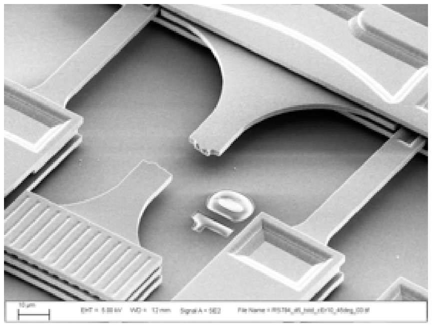


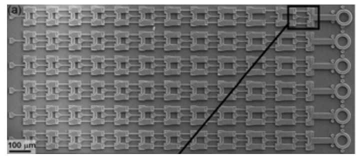
Weibull modulus for lifetime: $m_L = m/(n+1)$

n = exponent of Charles-Evans law for subcrit. crack growth

ZP Bažant, LJ Le, MZ Bazant (2009), Proc. National Acad.of Sciences 106, 11484--11489.

Metals on micrometer scale: same pdf, same size effect in Poly-Si MEMS devices as concrete on meter scale

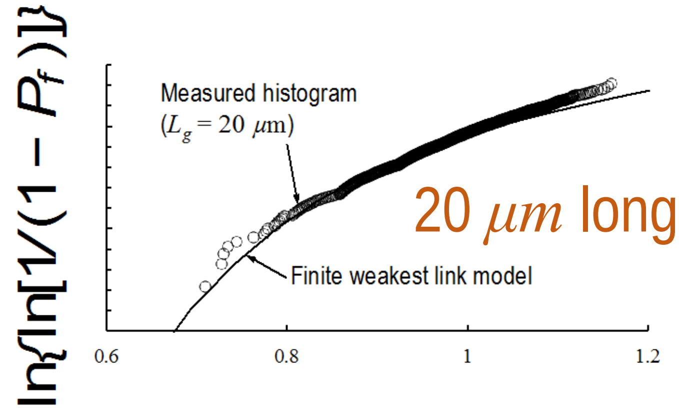




On-chip and slack-chain testers

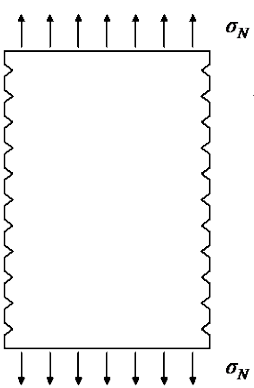
(Sandia, courtesy of B. Boyce)

Size effect on histograms



Finite weakest link model:

$$P_f = 1 - [1 - P_1(\sigma_N)]^n$$

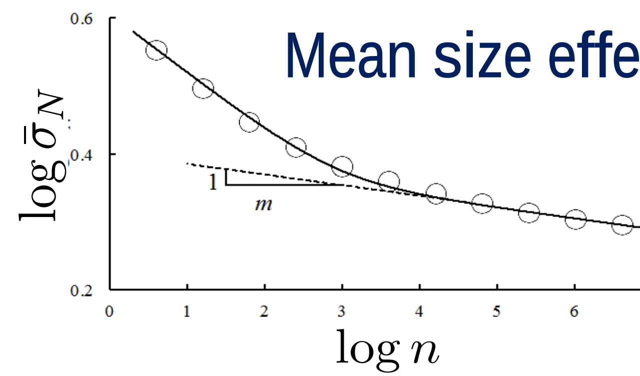


$$P_1(\sigma_N) = \int_0^\infty F_{f_t}(x\sigma_N) f_s(x)\,\mathrm{d}x$$

Random tensile strength

Random stress field

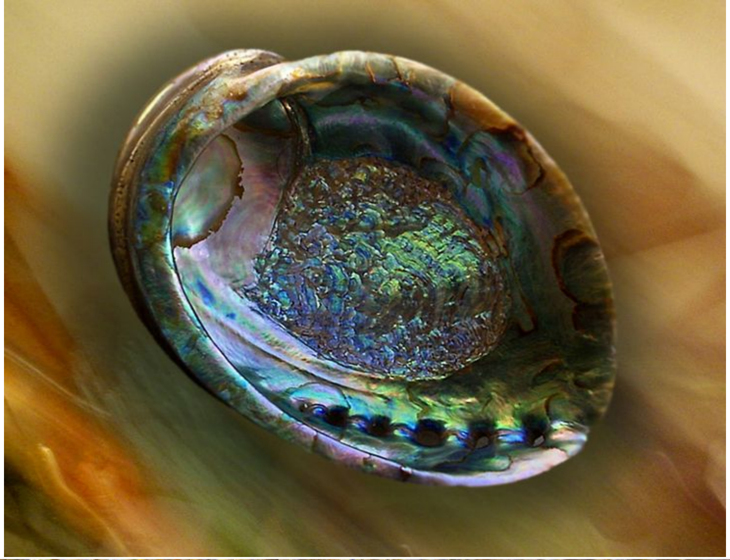
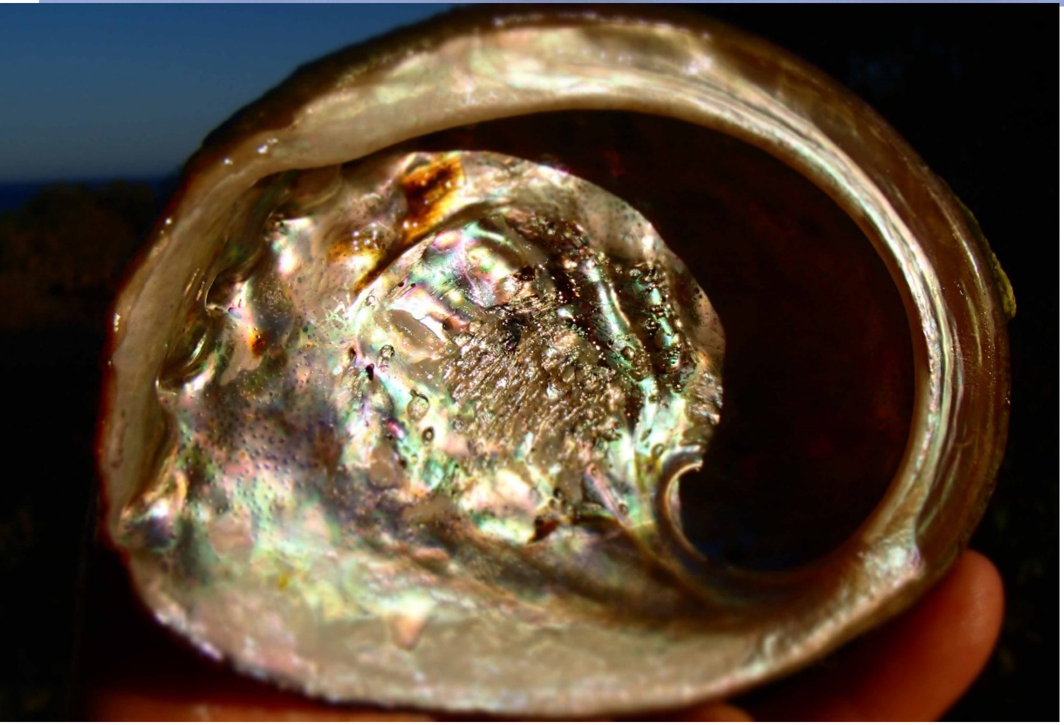
Mean size effect



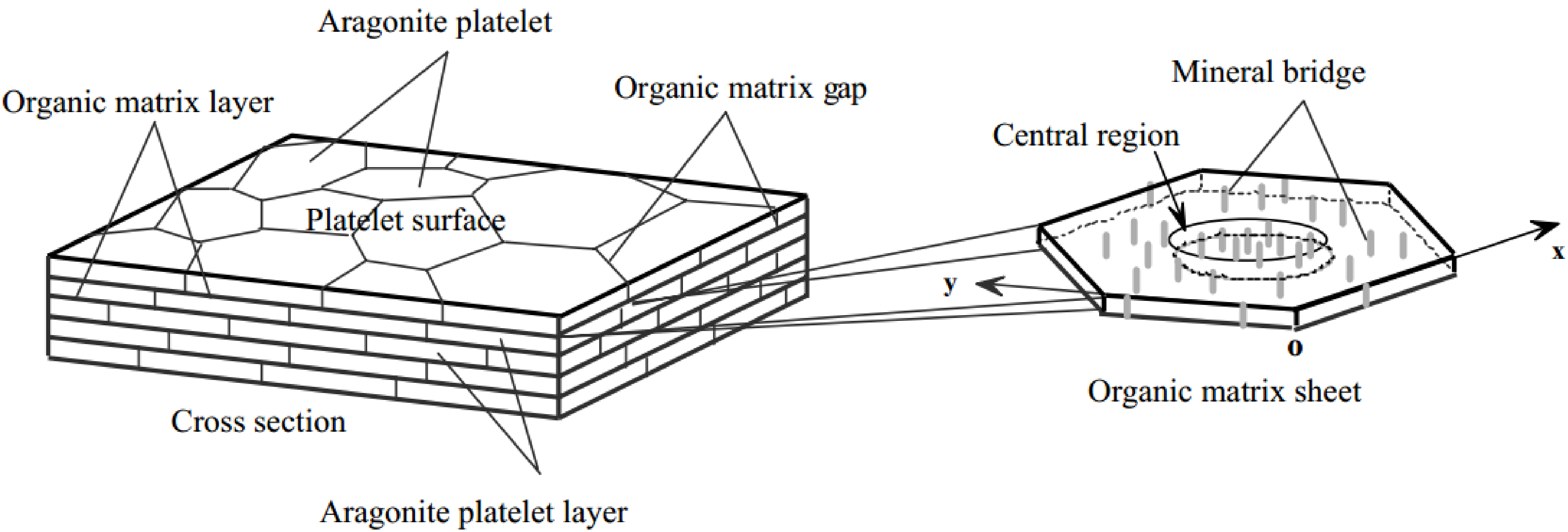
Le, Ballarini and Zhu (2015), *J. Amer. Cer. Soc.*

II.
Tail Strength Probability of Biomimetic Architectured Nacreous Materials:

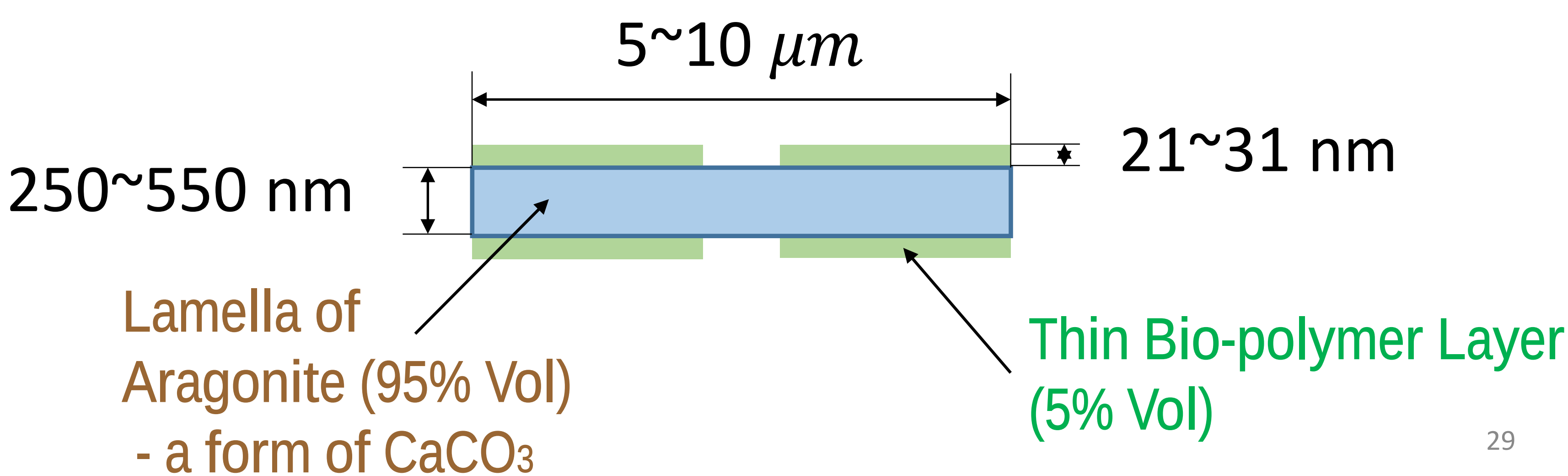
FISHNET STATISTICS


Nacre’s Nanostucture

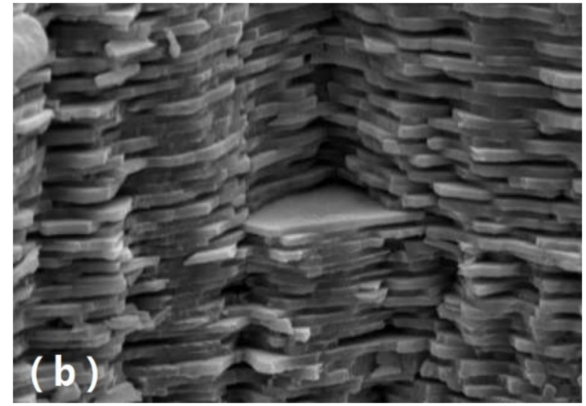


F. Song et al. / Biomaterials 24 (2003) 3623–3631

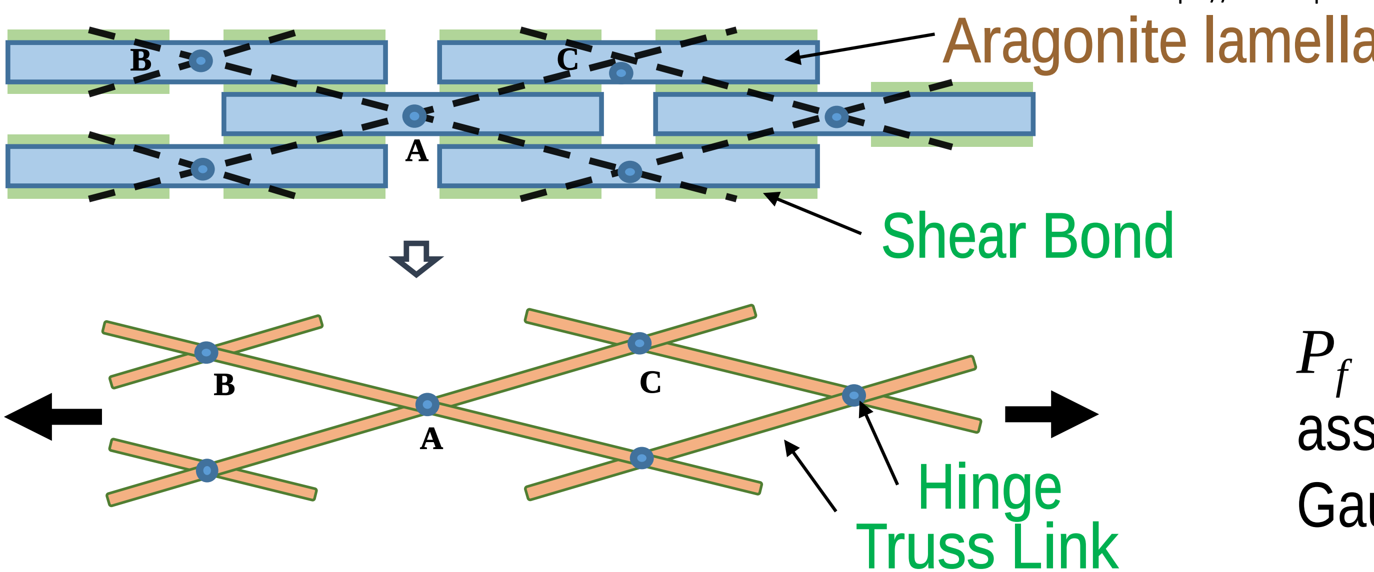


Idealization of Nacreous Nano-Architecture

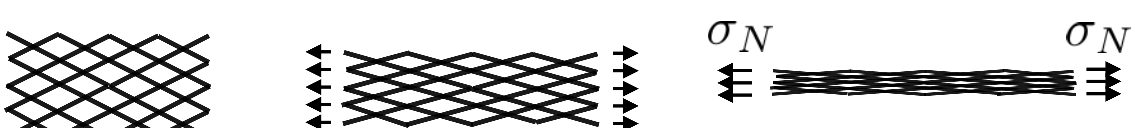
https://en.wikipedia.org/wiki/Nacre



https://en.wikipedia.org/wiki/Nacre

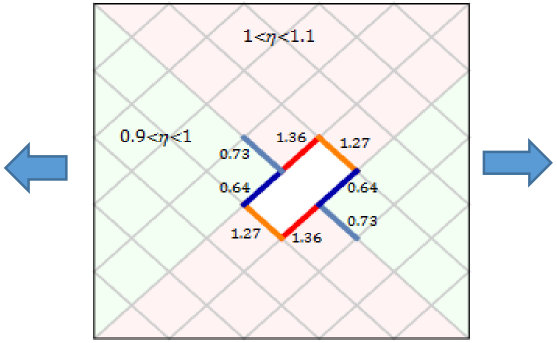


P_f of links is assumed to follow Gauss-Weibull graft



No Load Mechanism Collapsed (for computations)

New way to look at failure probability



(b)

— Prob. of survival:
Union of disjoint sets → a sum:

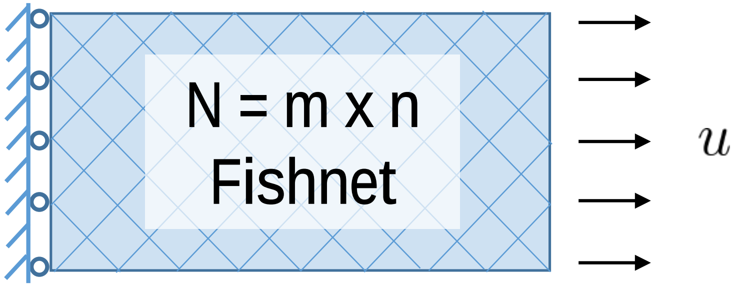
increasing CoV of Strength – more terms

$$1 - P_f(\sigma) = P_{S_0}(\sigma) + P_{S_1}(\sigma) + P_{S_2}(\sigma) + \cdots$$

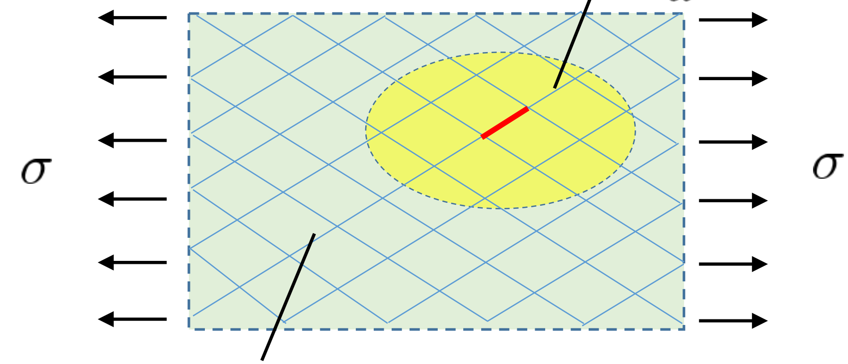
$$(P_{S_0} \gg P_{S_1} \gg P_{S_2})$$

with 0 failed link   with 1 failed link   with 2 failed links

The first term represents the weakest-link statistics



Two-Term Fishnet Model



Stress redIstributIon: solved via Laplace equation

one link

$$P_{S_0}(\sigma) = [1 - P_1(\sigma)]^N$$ = joint prob. of survival (intersection of sets)

$$P_{S_1}(\sigma) \simeq N P_1(\sigma) \cdot [1 - P_1(\sigma)]^{N-\nu_1-1} \cdot [1 - P_1(\eta_a^{(1)}\sigma)]^{\nu_1}$$

Any one of N links must already have failed

survival of remaining N-1 links

joint probability

equivalent redistributed stress

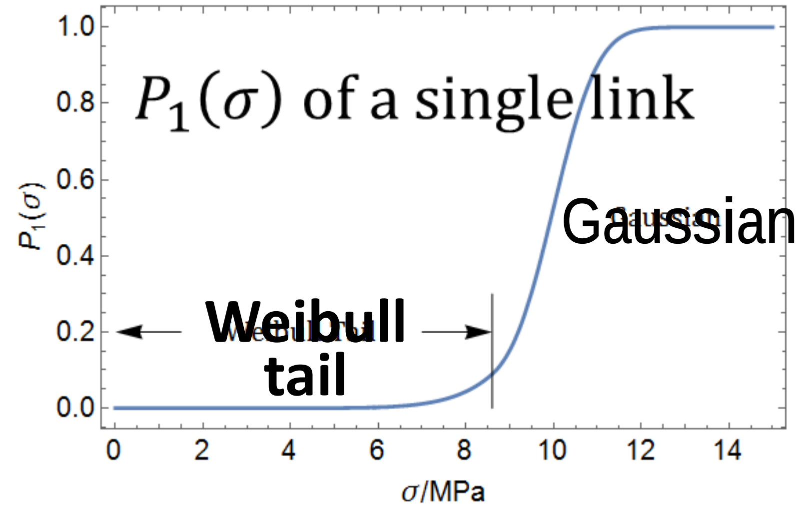
W Luo, ZP Bazant, 2017. Fishnet Statistics for Strength Scaling of Nacre-Like Imbricated Laminar Materials. JMPS, p.264
In brief: PNAS 2017, 12900

Three-Term Fishnet Model

No longer negligible when the **tail** of P_1 is **thick** or **CoV high**

$$1 - P_f(\sigma) = P_{S_0}(\sigma) + P_{S_1}(\sigma) + P_{S_2}(\sigma)$$

Same as before



2 cases: $P_{S_2} = P_{S_{21}} + P_{S_{22}}$

The second failure, if adjacent to the first one

if not

N and ν_1 independent choices

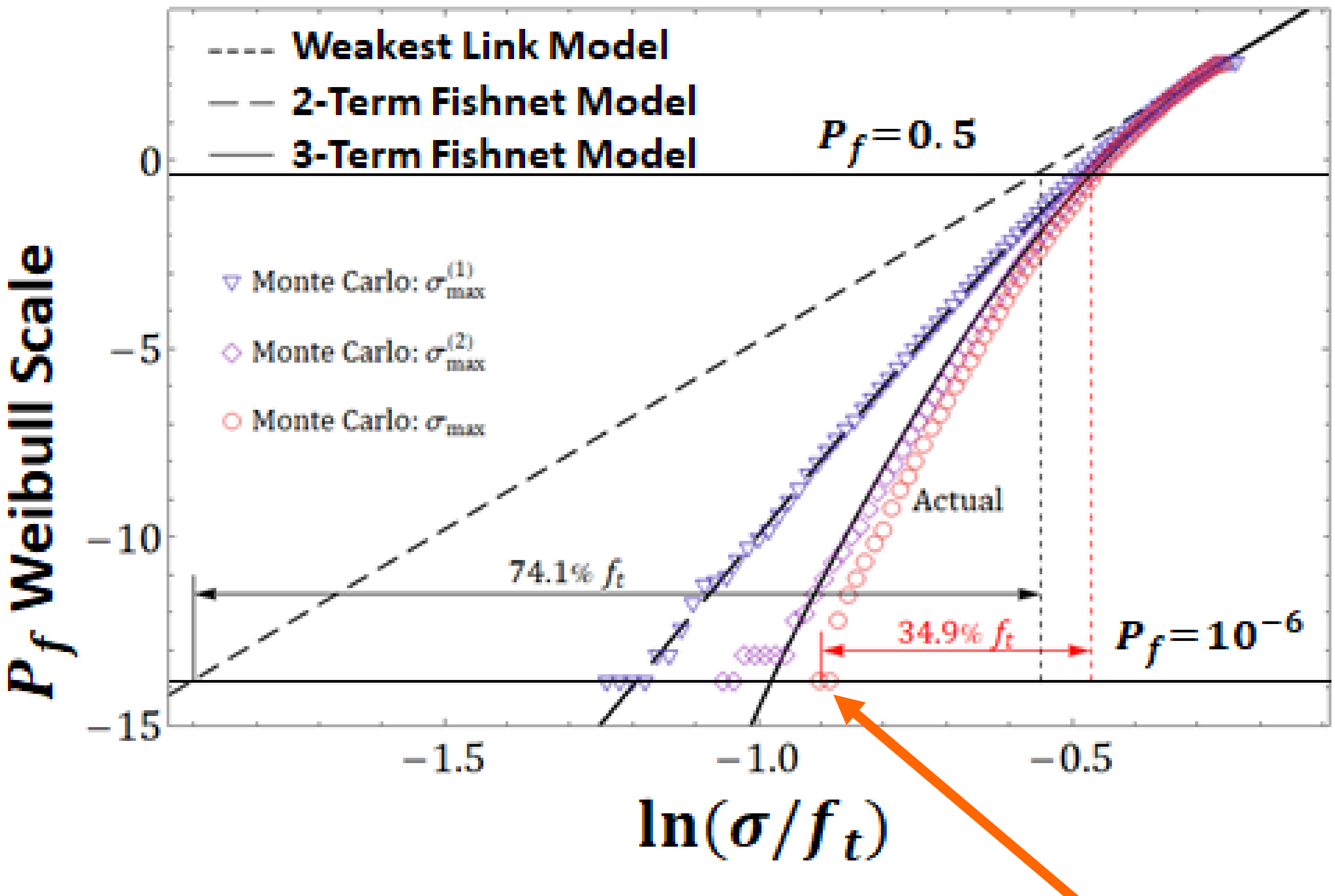
failure of two links — joint prob. with survival of remaining links

$$P_{S_{21}} = N\nu_1 \left\{ P_1(\sigma) P_1(\eta_b^{(1)}\sigma) - \frac{1}{2}[P_1(\sigma)]^2 \right\} \cdot [1 - P_1(\sigma)]^{N-\nu_2-2} \cdot [1 - P_1(\eta^{(2)}\sigma)]^{\nu_2}$$

$$P_{S_{22}} = \frac{1}{2} N(N - \nu_1 - 1)[P_1(\sigma)]^2 \cdot [1 - P_1(\sigma)]^{N-2\nu_1-2} \cdot [1 - P_1(\eta^{(2)}\sigma)]^{2\nu_1}$$

$P_{S_{22}} \gg P_{S_{12}}$: second damage occurring **far away** from the first

W Luo, ZP Bažant, 2017 Fishnet Statistics for Strength Scaling of Nacre-Like Imbricated Laminar Materials, PNAS & JMPS



NOTE: Enormous safety gain at tail

Enormous effect of fishnet architecture on safety:

	ln σ	σ /MPa	P_f
Weakest-Link Model	1.8	6.05	29.5×10^{-6}
Two-Term Fishnet	1.8	6.05	1.19×10^{-6}

— 25-fold decrease of failure probability P_f at cdf tail!

(and more for more terms and higher scatter)

W Luo, ZP Bažant, 2017. Fishnet Statistics for Strength Scaling of Nacre-Like Imbricated Laminar Materials. Arxiv