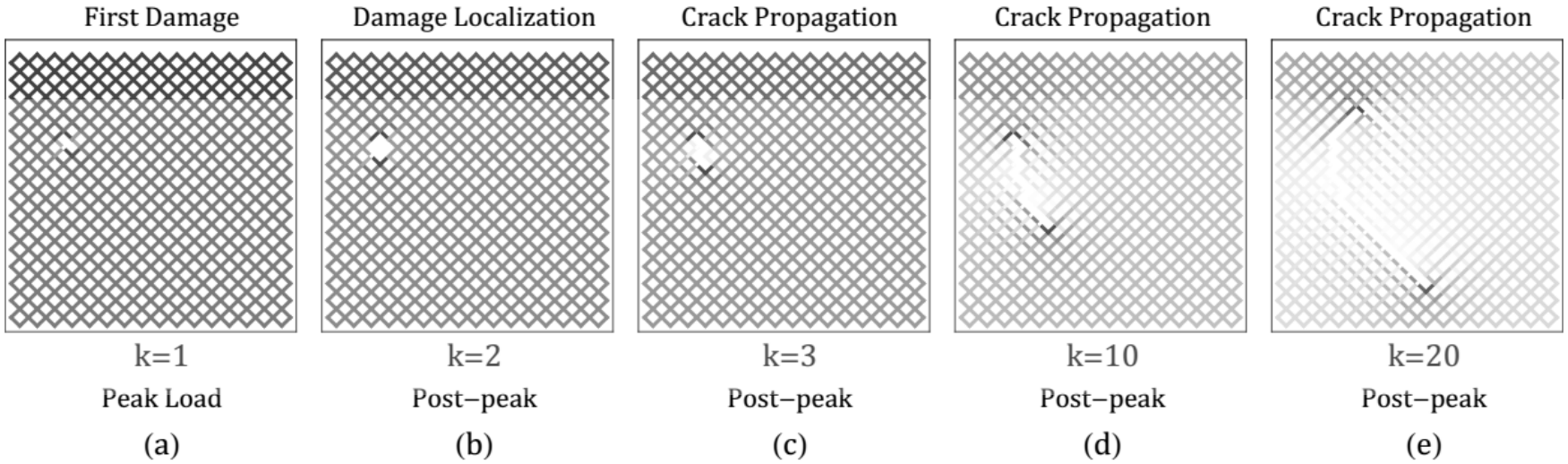
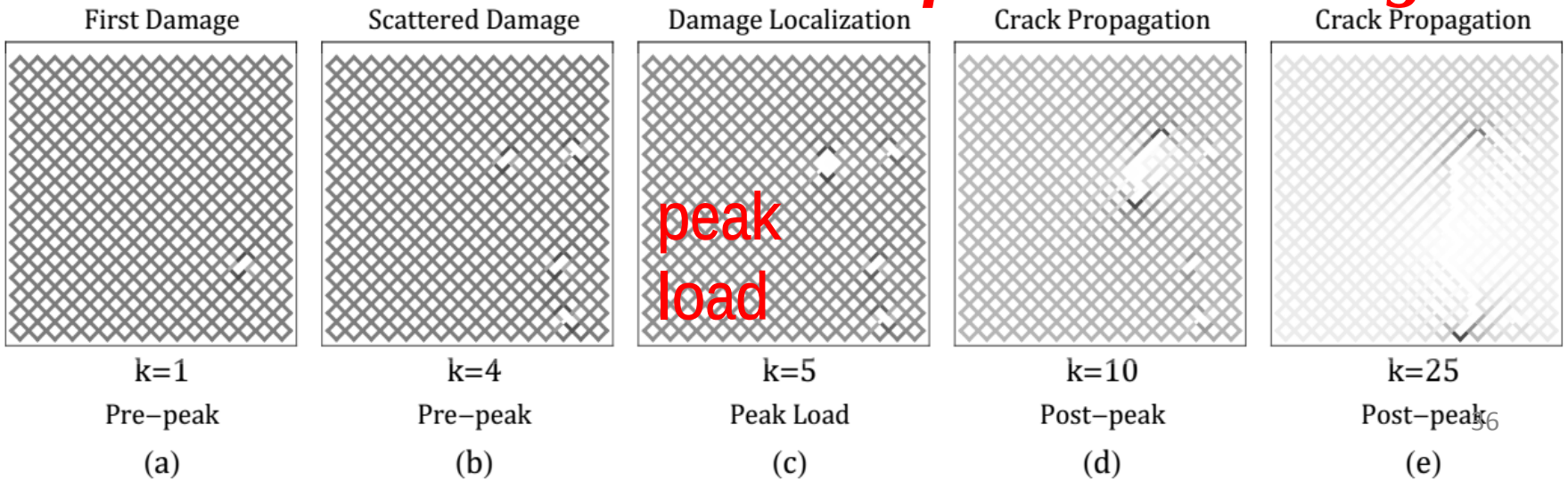
Effect of CoV of Link Strength Scatter at 10^{-6} Tail

Low scatter – one crack – close to weakest link …



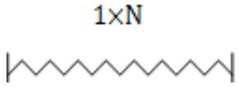
High scatter – many cracks – not close

→ *safer! Scatter is good!*

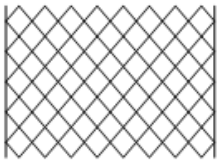


Weibull to Gaussian cdf Transition upon Changing Aspect ratio of Fishnet

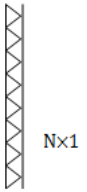
Chain



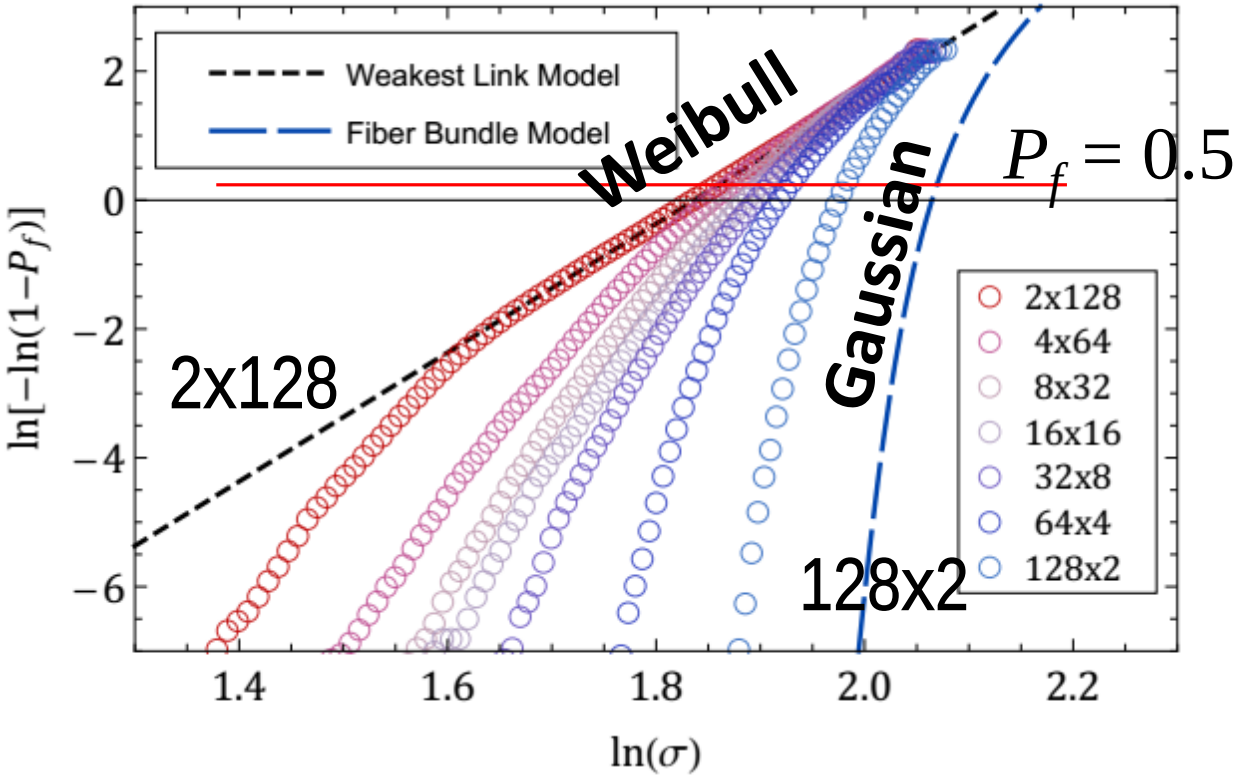
Fishnet



Bundle



Upper Bound→ **Increasing Reliability→** **Lower Bound**



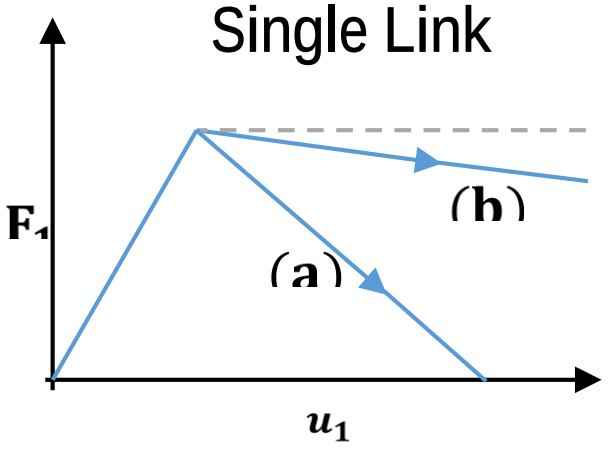
W Luo, ZP Bazant. Fishnet Statistics for Strength Scaling of Nacre-Like Imbricated Laminar Materials. JMPS 2017

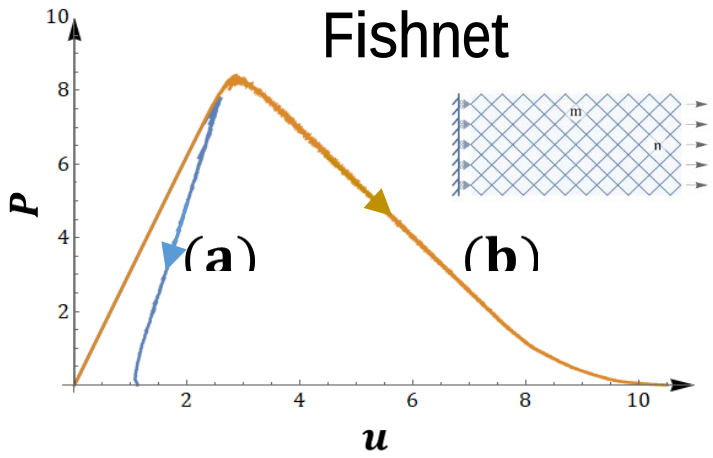
III.
Latest Results at NU on Fishnet Statistics

(in detail, see poster of Wen Luo)

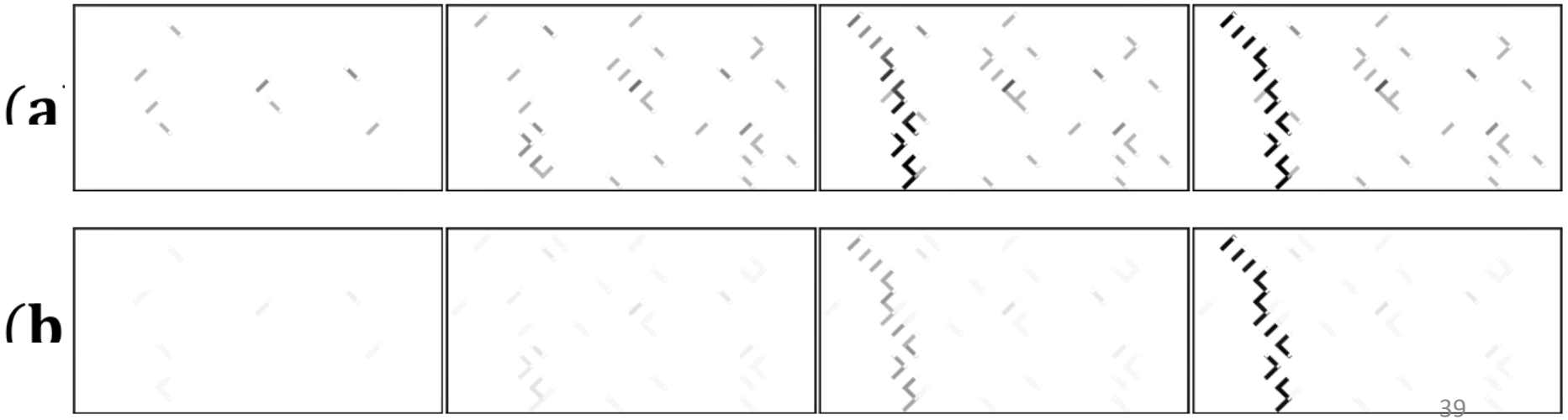
Modification for gradually softening links:

series of stress drops





Fishnet Damage Evolution

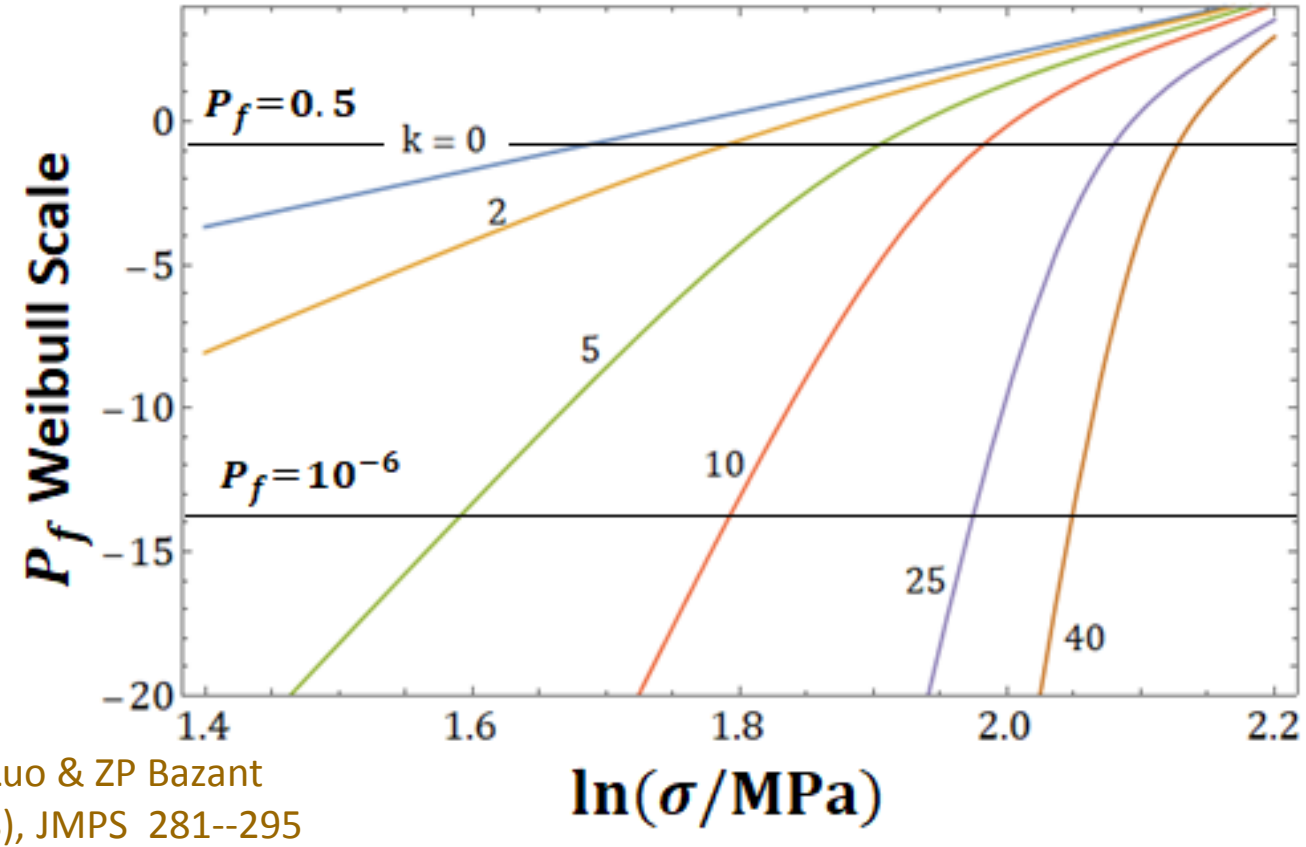


Let N_c = number of damaged links at max. load

$$P_f(x) = \mathbb{P}(\sigma_{max} \le x) = \sum_{k=0}^{N} \mathbb{P}(N_c = k)\,\mathbb{P}(\sigma_{max} \le x \mid N_c = k)$$

Distribution of k^{th} smallest minimum, $s_{(k)}$, of link strength:

Based on Order Statistics: $W_k(x) = \mathbb{P}[s_{(k)} \le x]$



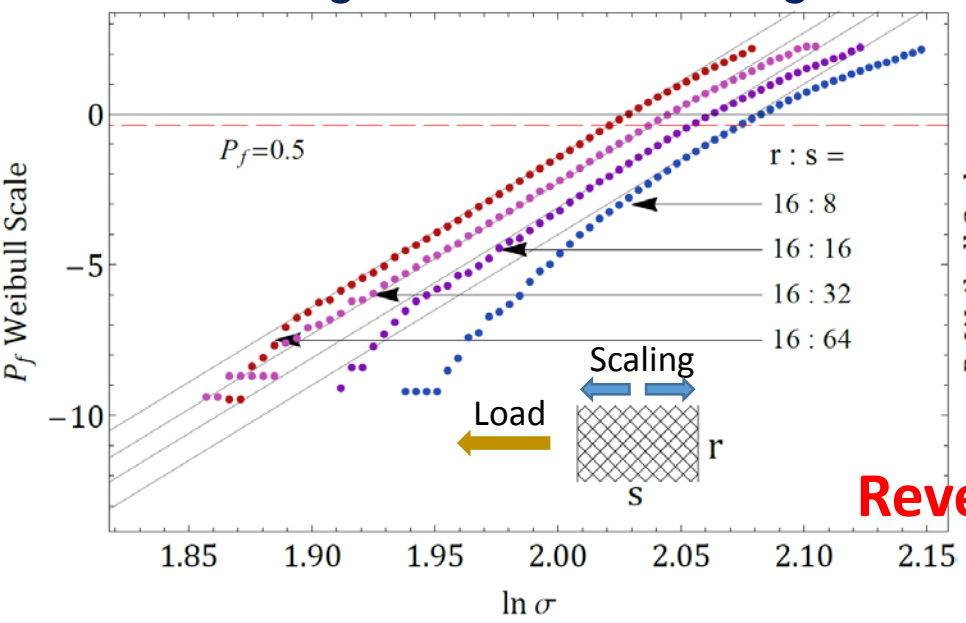
Random cluster of damages: N_c follows geometric Poisson distribution (Pólya-Aeppli)

Wen Luo & ZP Bazant (2018), JMPS 281--295

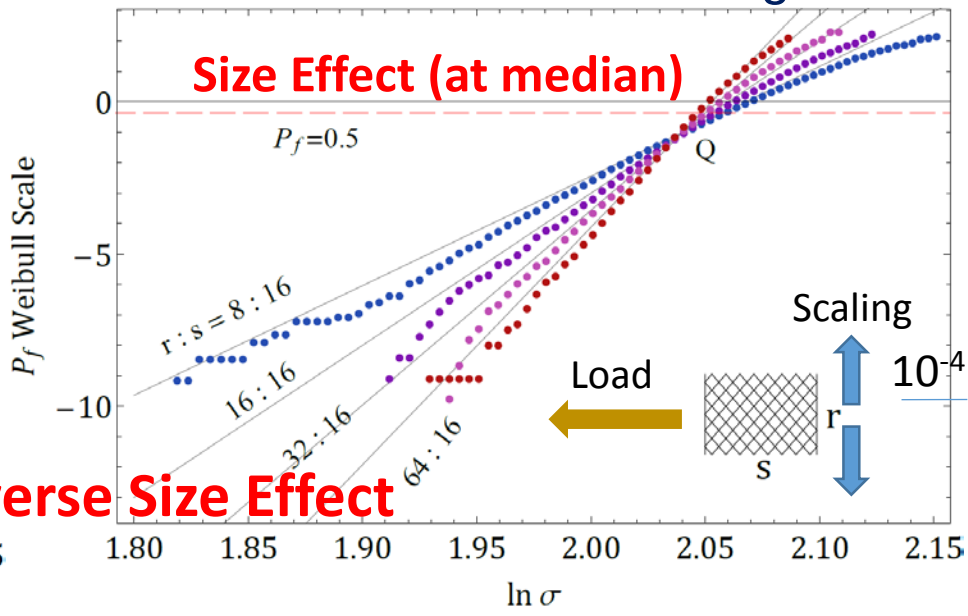
Size Effect = Joint Effect of Horizontal and Vertical Scaling

Simulated Sample Size = 10^4

Longitudinal Scaling



Transverse Scaling



Reverse Size Effect

Longitudinal scaling:

Weakest-link rule – the histogram shifts up by $\ln(s_2/s_1)$ if the length is increased from s_1 to s_2;

Transverse scaling:

Histograms rotate about a point, Q, at a constant rate, equally for each doubling of width

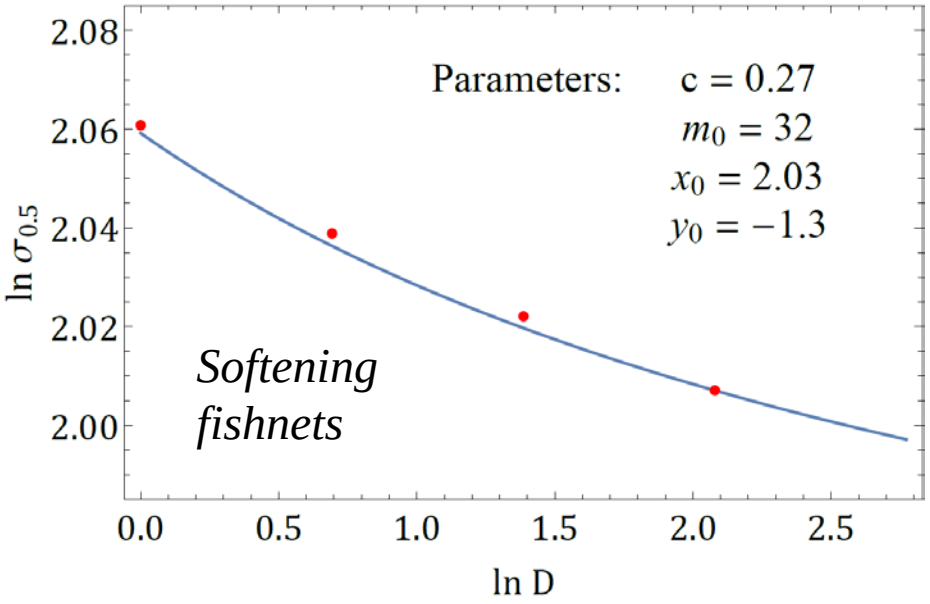
→ **Slope increases**.

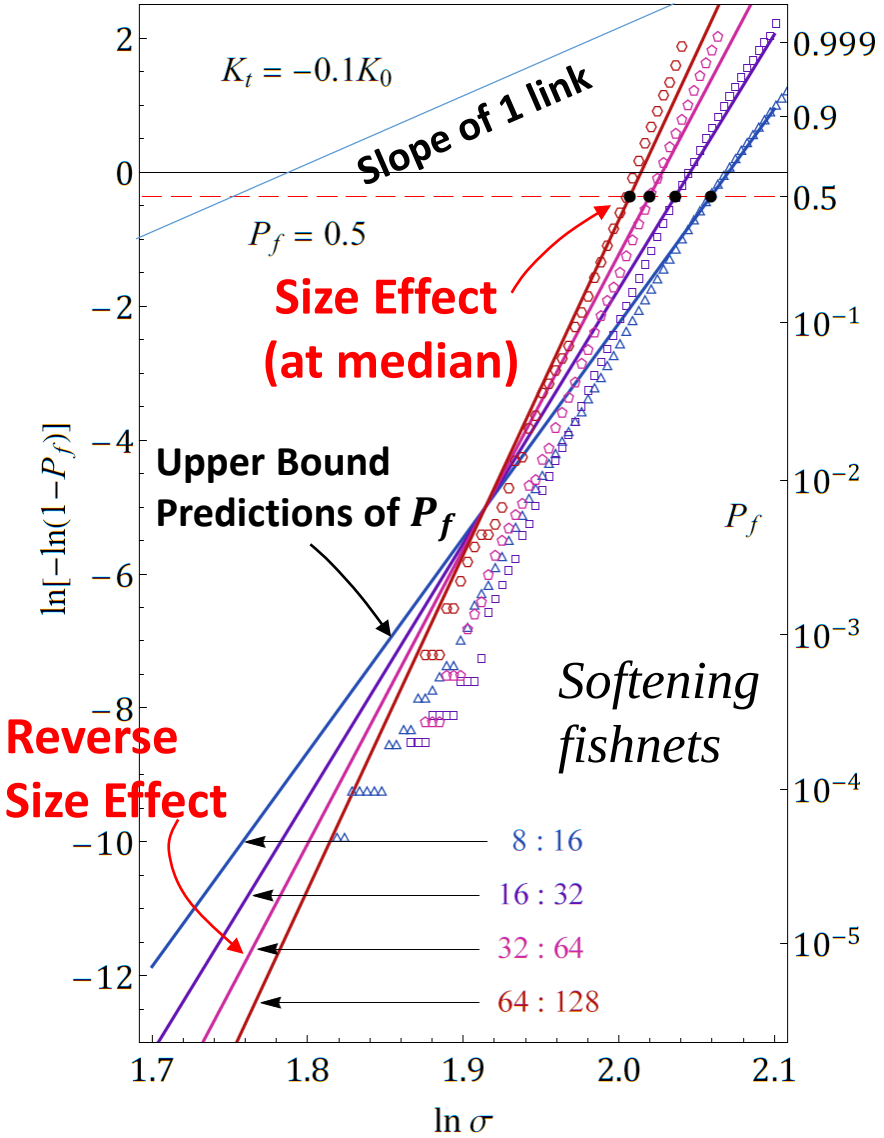
Inferring Strength Distribution from Size Effect

Strength Distribution: $Y - y_0 = m_0[1 + c\ln(r/r_0)](X - x_0) + \ln(s/s_0)$

Median Size Effect:

$$\ln \sigma_{0.5} = \frac{\ln\ln 2 - y_0 - \ln D}{m_0(1 + c\ln D)} + x_0$$





To sum up:

- For quasibritle materials, we need

  TAIL-RISK DESIGN

  (not just Mean & Standard Deviation)

- The safety factor is size dependent.

- The reliability indices (Cornell, Hasofer-Lind) have been modified.

For quasibrittle materials, and esp. architectured and biomimetic ones:

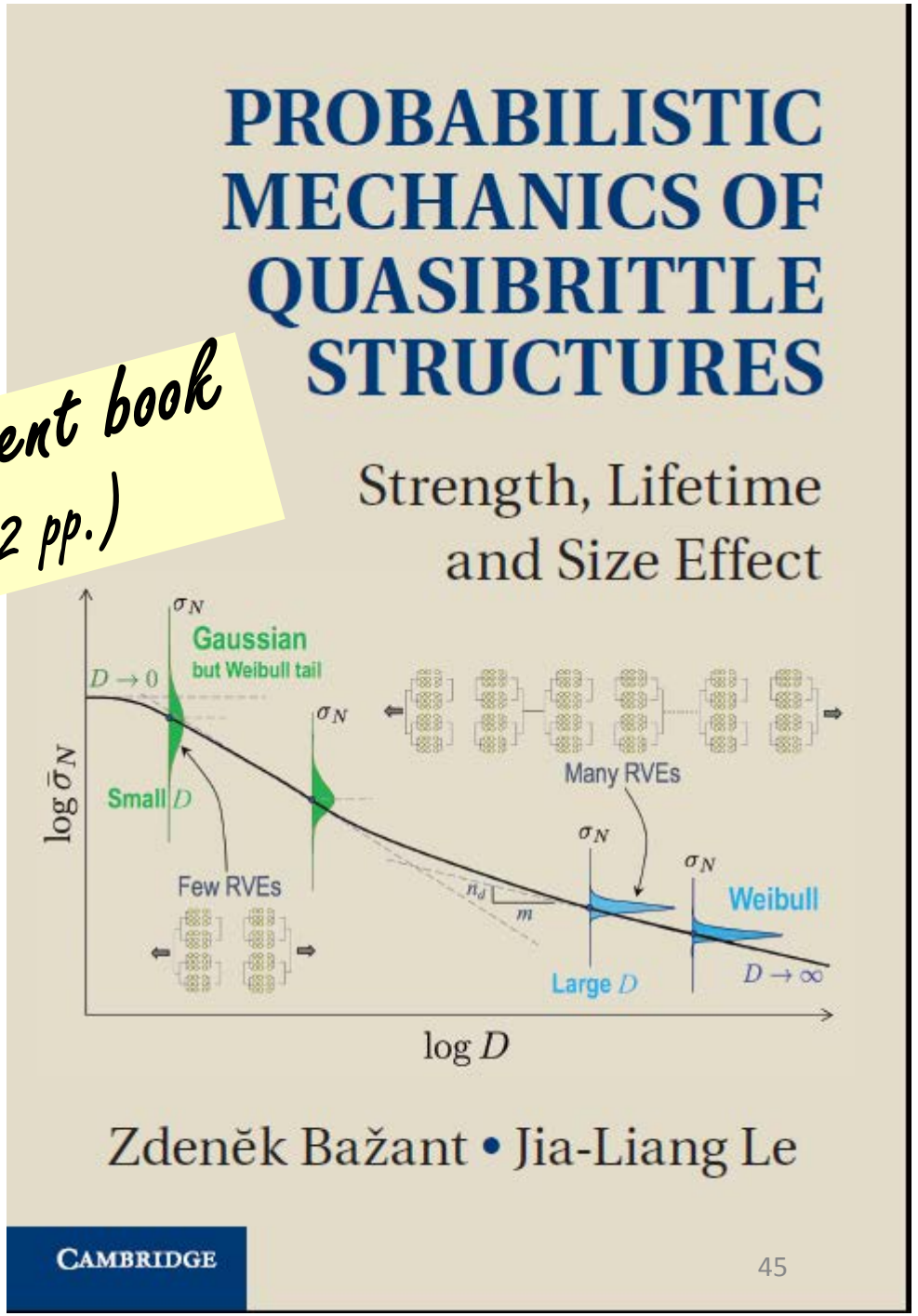
|Error| in safety factors >> |Error| in computational mechanics

because the devil is in the tail

Thanks for Listening!

Questions?

Recent book (322 pp.)



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