Effect of CoV of Link Strength Scatter at $10^{-6}$ Tail

**Low scatter** – one crack – close to weakest link …

- First Damage
- Damage Localization
- Crack Propagation

**High scatter** – many cracks – not close

$\rightarrow$ safer! Scatter is good!

- First Damage
- Scattered Damage
- Damage Localization
- Crack Propagation

- k=1
  - Peak Load
  - Pre-peak
  - (a)

- k=4
  - Pre-peak
  - (b)

- k=5
  - Peak Load
  - (c)

- k=10
  - Post-peak
  - (d)

- k=20
  - Post-peak
  - (e)
Weibull to Gaussian cdf Transition upon Changing Aspect ratio of Fishnet

Chain

Fishnet

Bundle

Upper Bound ➔ Increasing Reliability ➔ Lower Bound

Chain Bundle Fishnet

Increasing Reliability

Weibull

Gaussian

$P_f = 0.5$

III. Latest Results at NU on Fishnet Statistics

(in detail, see poster of Wen Luo)
Modification for **gradually softening links**: series of stress drops

**Single Link**

- $F_s$
- $u_1$

**Fishnet**

- $P$
- $u$

**Fishnet Damage Evolution**

- (a)
- (b)
Let $N_c$ = number of damaged links at max. load

$$P_f(x) = \mathbb{P}(\sigma_{max} \leq x) = \sum_{k=0}^{N} \mathbb{P}(N_c = k) \mathbb{P}(\sigma_{max} \leq x \mid N_c = k)$$

Distribution of $k^{th}$ smallest minimum, $s_{(k)}$, of link strength:

**Based on Order Statistics:**

$$W_k(x) = \mathbb{P}[s_{(k)} \leq x]$$

Random cluster of damages: $N_c$ follows geometric Poisson distribution (Pólya-Aeppli)
Size Effect = Joint Effect of Horizontal and Vertical Scaling
Simulated Sample Size = $10^4$

**Longitudinal Scaling**

- **Weakest-link rule** – the histogram shifts up by $\ln(s_2/s_1)$ if the length is increased from $s_1$ to $s_2$;

**Transverse Scaling**

- **Histograms rotate** about a point, $Q$, at a constant rate, equally for each doubling of width → **Slope increases**.
Inferring Strength Distribution from Size Effect

Strength Distribution: \( Y - y_0 = m_0[1 + c \ln(r/r_0)](X - x_0) + \ln(s/s_0) \)

Median Size Effect:

\[
\ln \sigma_{0.5} = \frac{\ln \ln 2 - y_0 - \ln D}{m_0(1 + c \ln D)} + x_0
\]

Parameters: 
- \( c = 0.27 \)
- \( m_0 = 32 \)
- \( x_0 = 2.03 \)
- \( y_0 = -1.3 \)
To sum up:

- For quasibrittle materials, we need **TAIL-RISK DESIGN** (not just Mean & Standard Deviation)

- The safety factor is size dependent.

- The reliability indices (Cornell, Hasofer-Lind) have been modified.
For *quasibrittle* materials, and esp. architectured and biomimetic ones:

| Error | in safety factors |  >>  | Error | in computational mechanics |

*because the devil is in the tail*
Thanks for Listening!

Questions?


Recent book (322 pp.)

PROBABILISTIC MECHANICS OF QUASIBRITTLE STRUCTURES
Strength, Lifetime and Size Effect
Zdeněk Bažant • Jia-Liang Le