Instantaneous Homogenized Response

- Power-law behavior for the grains:

$$\phi_{(k)}(\tau) = \frac{\dot{\gamma}_0 (\tau_0)_{(k)}}{n+1} \left| \frac{\tau}{(\tau_0)_{(k)}} \right|^{n+1}$$

- Statistically isotropic distribution of the grains both in space and orientation implies:

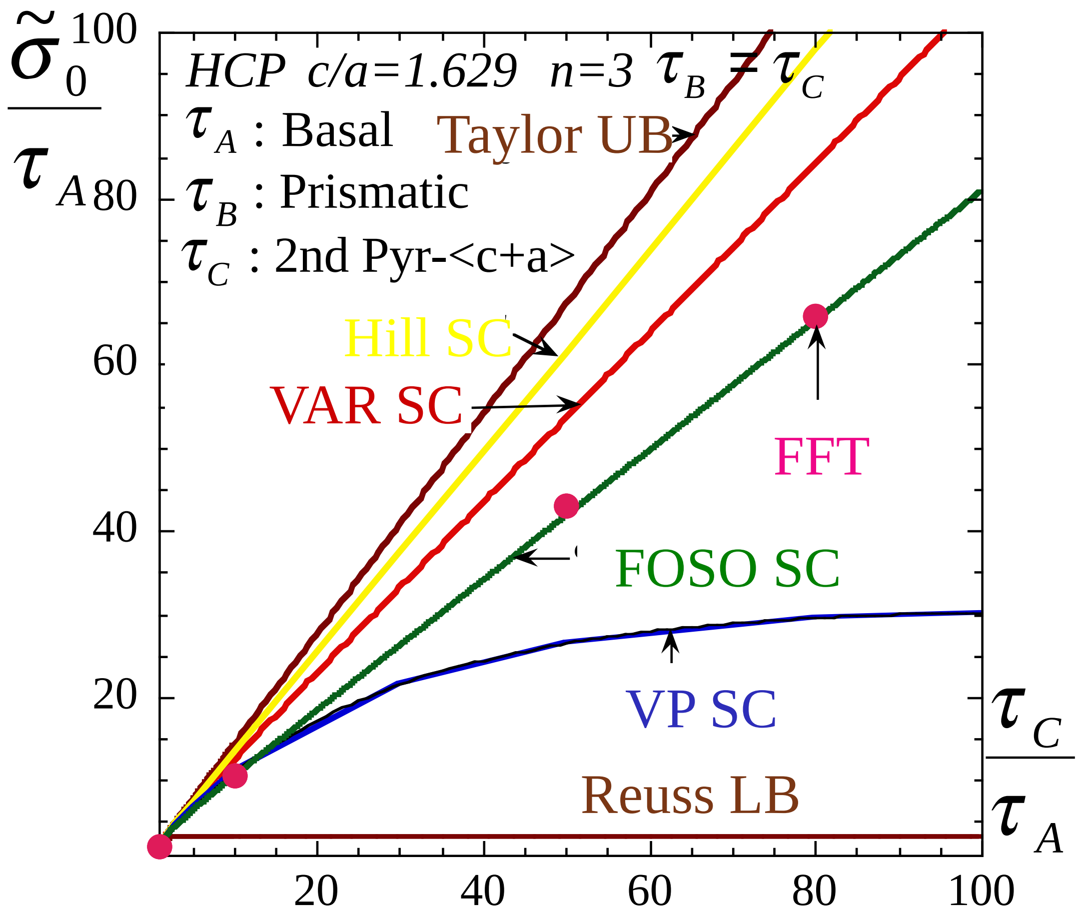
$$\widetilde{u}(\overline{\boldsymbol{\sigma}}) = \frac{\dot{\gamma}_0 \widetilde{\sigma}_0}{n+1} \left(\frac{\overline{\sigma}_e}{\widetilde{\sigma}_0} \right)^{n+1}$$

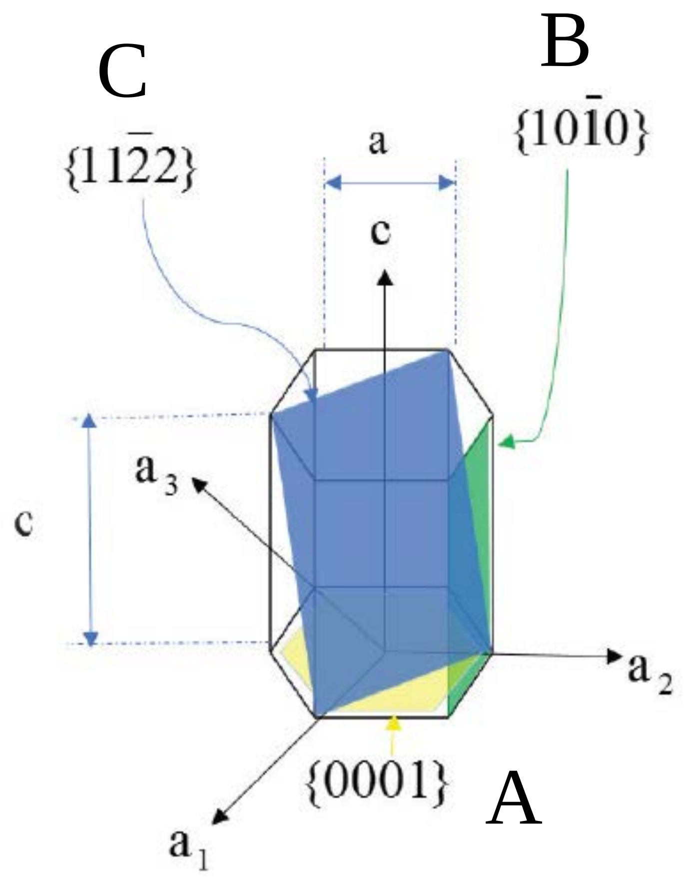
where $\overline{\sigma}_e$ is the von Mises equivalent stress,

and $\widetilde{\sigma}_0$ is the effective flow stress (depends on $\det \overline{\boldsymbol{\sigma}}$).

Ice Polycrystal

Effective Flow Stress





$n = 3$

Grain Anisotropy

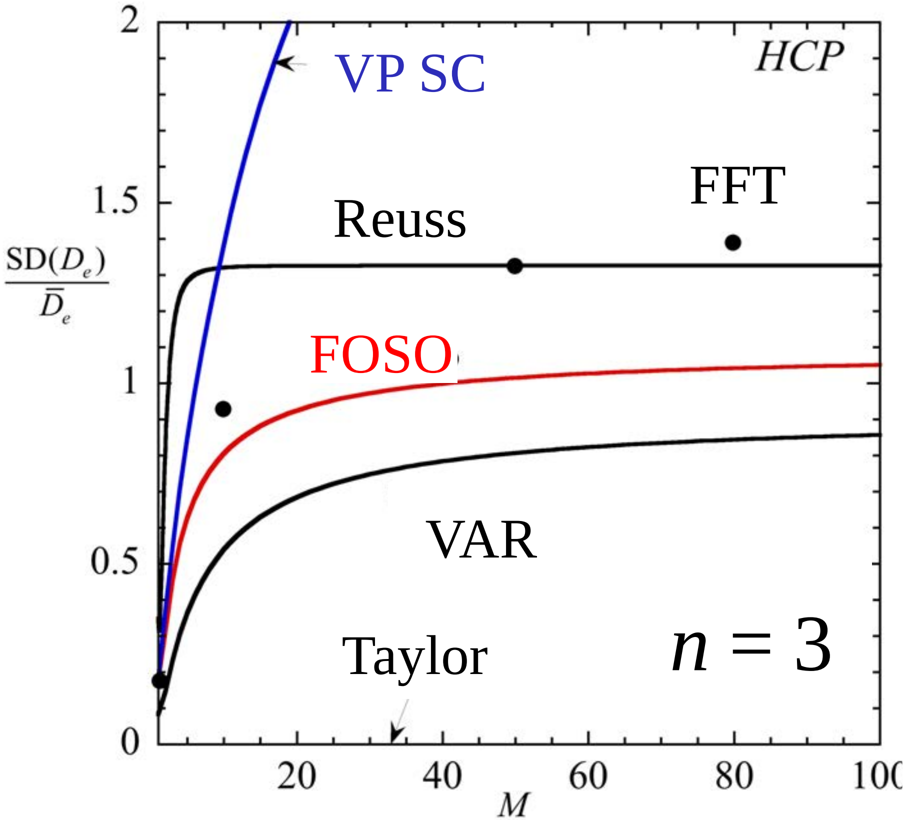
$M = \tau_B/\tau_A = \tau_C/\tau_A$

•FFT - Full numerical simulation

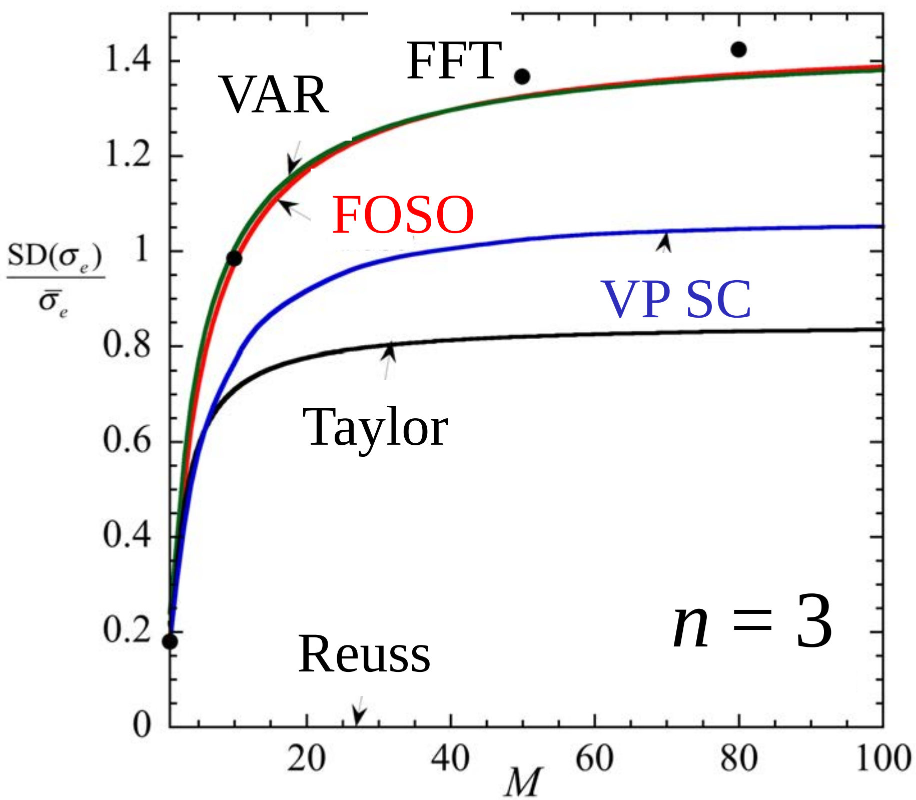
Lebensohn, Liu & PC (2004) AM

Ice Polycrystal

Standard Deviation of Equivalent Strain-Rate



Standard Deviation of Equivalent Stress

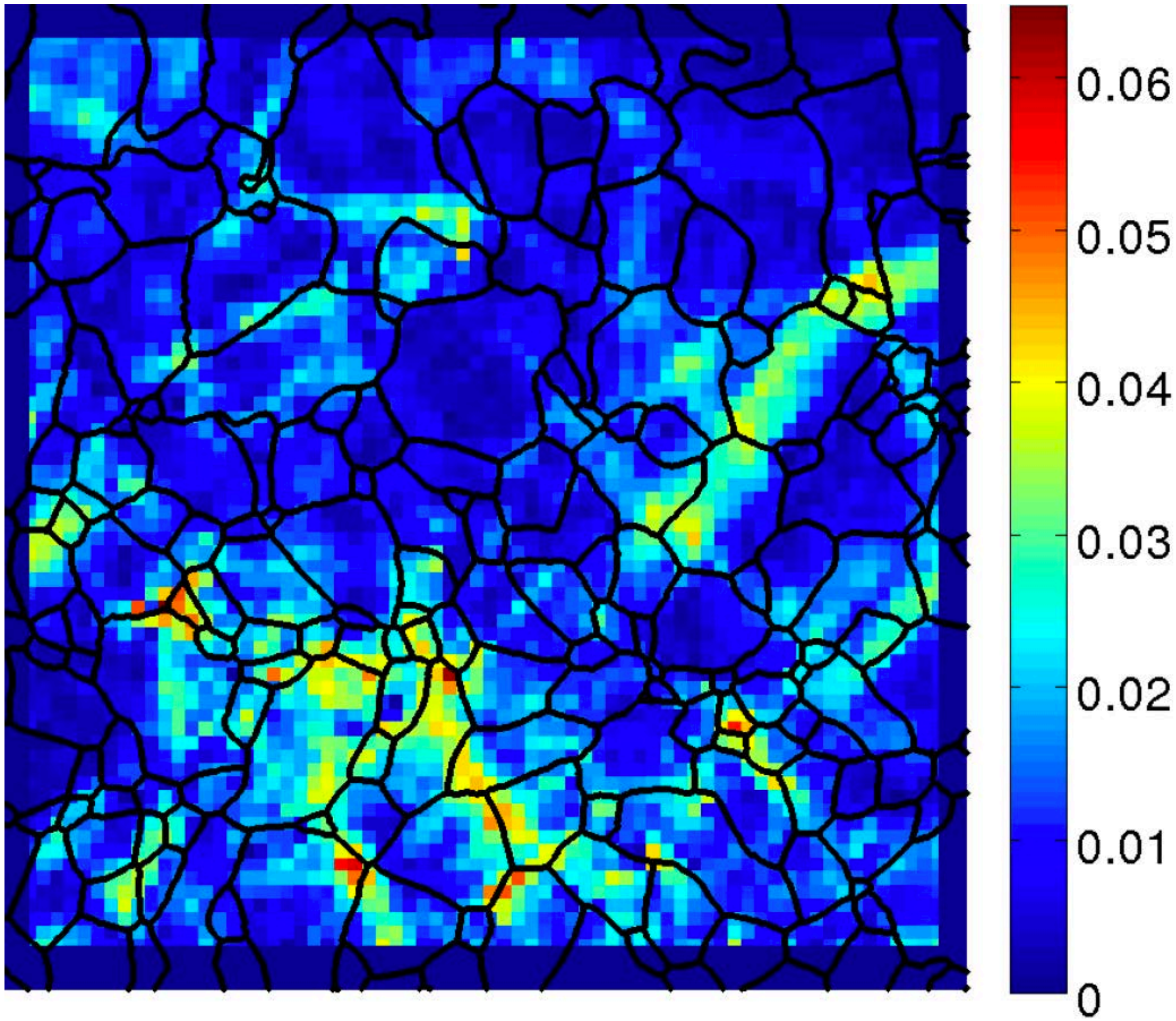


Grain Anisotropy

$$M = \tau_B/\tau_A = \tau_C/\tau_A$$

Ice Polycrystal

DIC: Map of Equivalent Strain

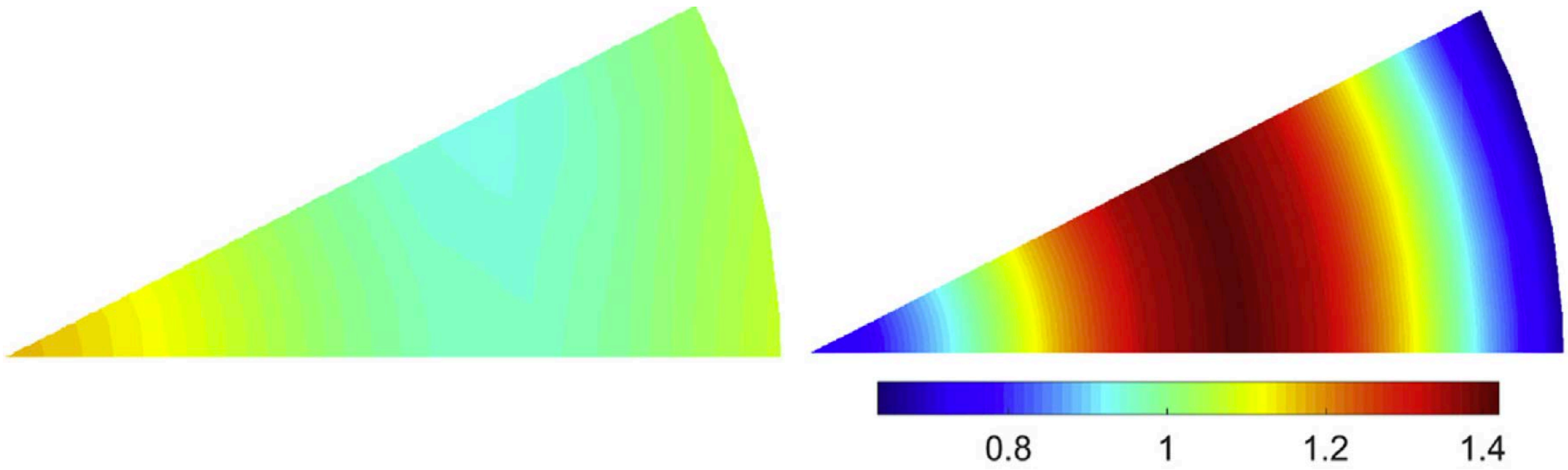


Grennerat et al. (2012) Acta Mat.

High-Anisotropy HCP Polycrystals

Grain Average Strain-Rate

Ice



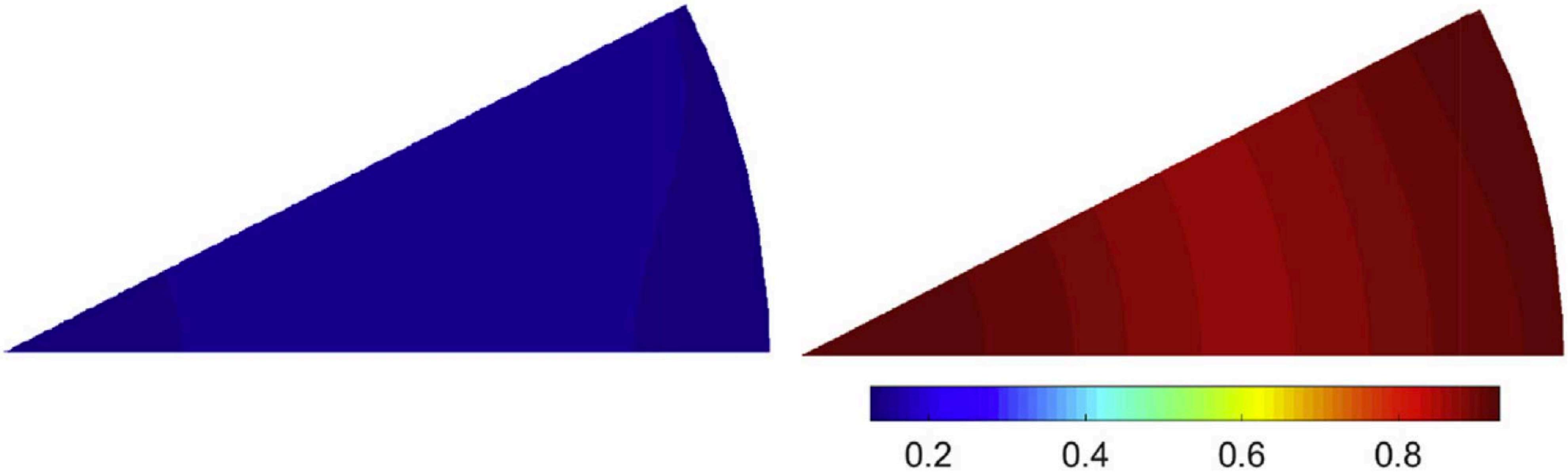
$M = 1$ $M = 60$

$n = 3$

High-Anisotropy HCP Polycrystals

Grain SD Strain-Rate Fluctuations

Ice



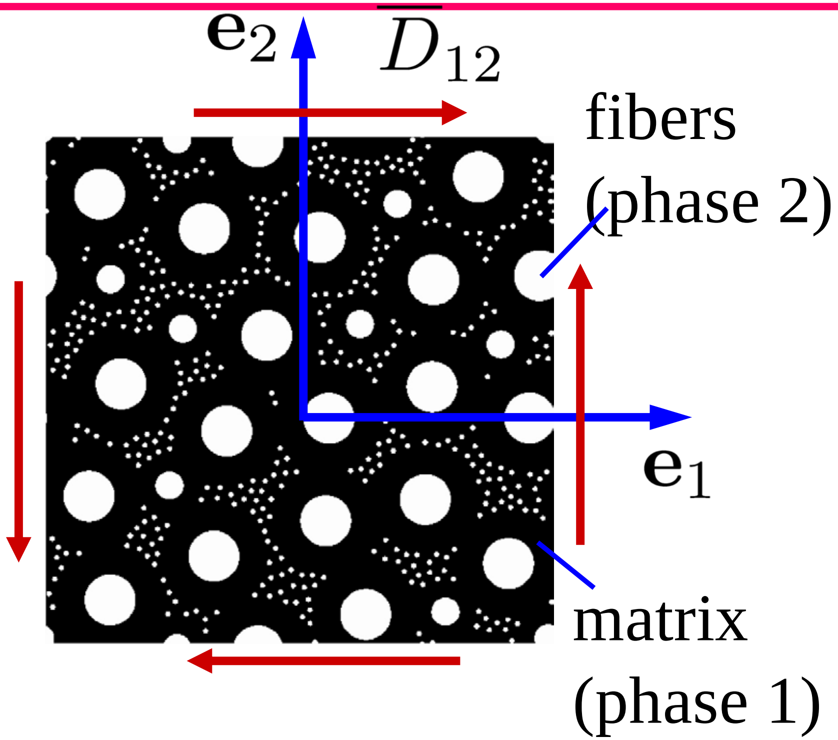
$M = 1$ $M = 60$

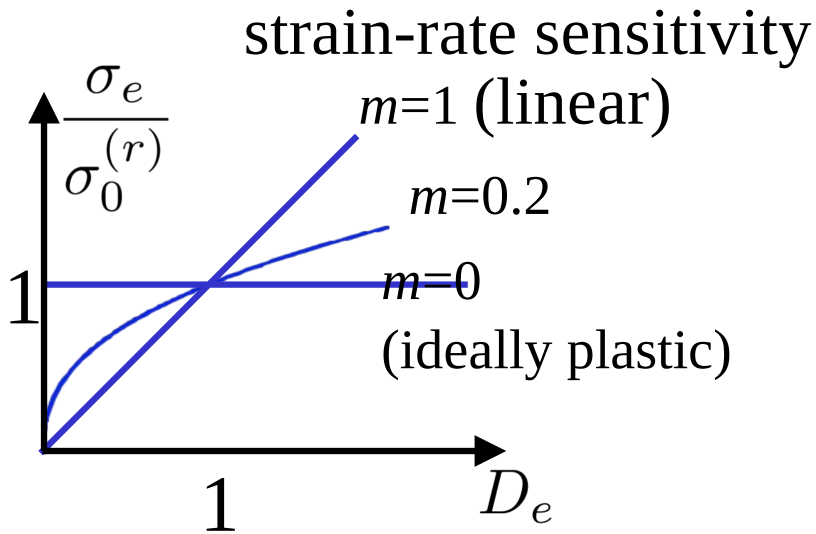
$n = 3$

APPLICATION: FIBER-REINFORCED COMPOSITES

Idiart, Moulinec, Ponte C, Suquet (2006) JMPS

Fiber-reinforced composites





- Distribution of fibers in the plane is random and isotropic
- Plane strain conditions
- Incompressible, power-law phases:

$$w^{(r)}(\boldsymbol{D}) = \frac{\sigma_0^{(r)}}{1+m} D_e^{1+m}$$

- In-plane effective behavior:

$$\widetilde{W}(\overline{\boldsymbol{D}}) = \frac{\widetilde{\sigma}_0}{1+m} \overline{D}_e^{1+m}$$

effective flow stress

$$\widetilde{\sigma}_0(m, c^{(r)}, \sigma_0^{(r)})$$

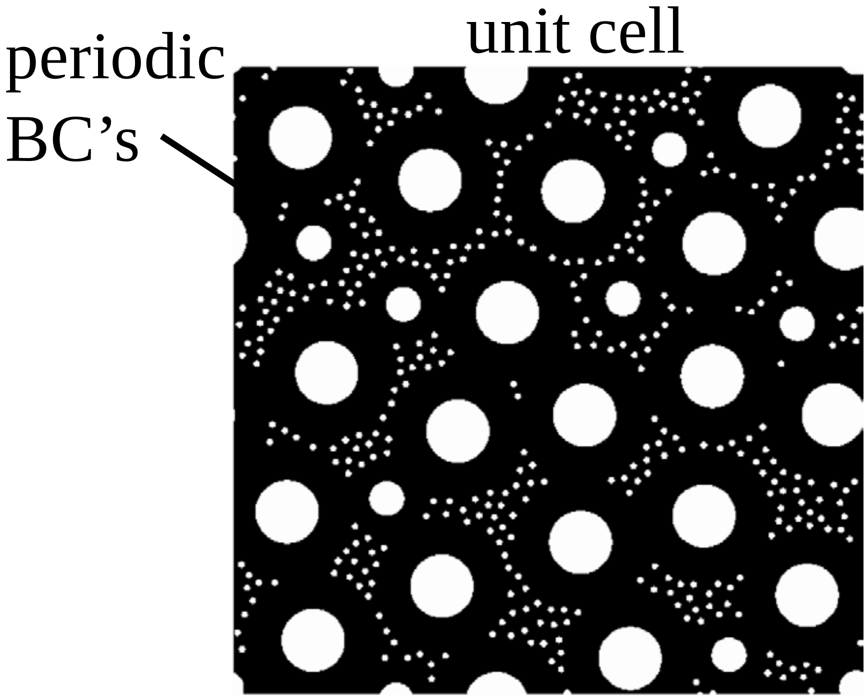
Fiber-reinforced composites

- Fast Fourier Transform (FFT) simulations

  - 490 composite cylinders of three different sizes

  - ensemble average over 20 configurations in order to obtain statistical homogeneity and isotropy

  - discretization: 1024 x 1024 pixels



Mean concentration of fibers $\approx$ 0.21

- Second-order estimates

  - make use of the Hashin-Shtrikman linear estimates (Willis 1978, Ponte Castañeda & Willis 1995)

  - require the solution of three nonlinear equations