Stochastic Modeling of Damage Localization in Quasibrittle Materials

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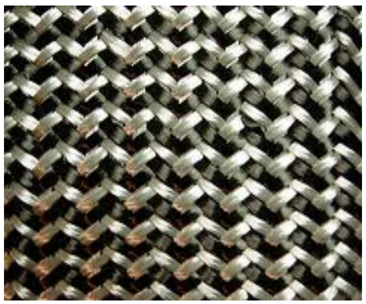
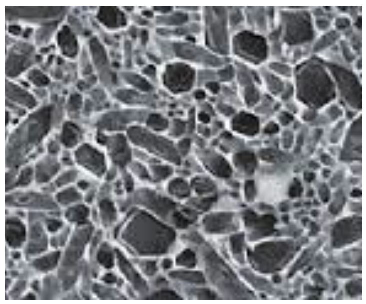
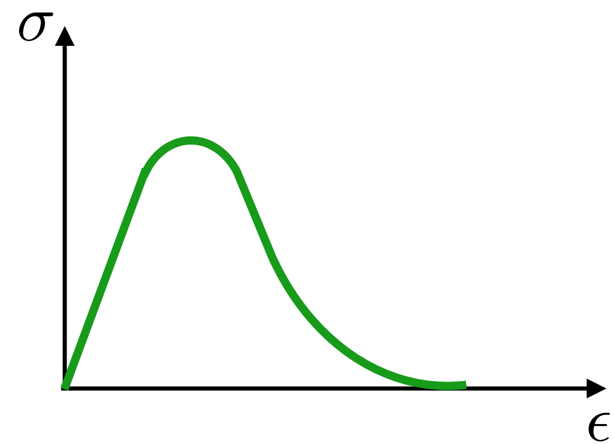
Background — Strain Localization in Quasibrittle Materials

Quasibrittle (brittle heterogenous) materials exhibit a softening stress-strain behavior, which leads to strain localization.


composites


ceramics


concrete



$$\Delta(\partial \dot{u}_i/\partial x_j) = g_i(x_1)\delta_{j1}$$

Localization instability:
— non-uniform deformation incipient in a band with continuing equilibrium and homogenous deformation outside the band.

—Eigenvalue analysis of acoustic tensor (e.g. Rudnicki and Rice 1976 — finite strain plasticity, Jirásek 2007 — damage softening)

Background — Localization Limiters

Strain localization leads to the issue of mesh objectivity in continuum FE modeling — loss of ellipticity of the governing equation.

Existing numerical techniques to treat strain localization — localization limiters

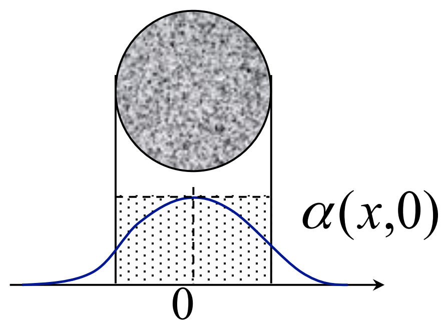
I. Crack band model (Bažant and Oh 1983)

— adjust the post-peak of stress-strain curve to preserve the fracture energy (link smeared damage to cohesive crack)

II. Nonlocal continuum

— introduce non-locality to the constitutive relationship

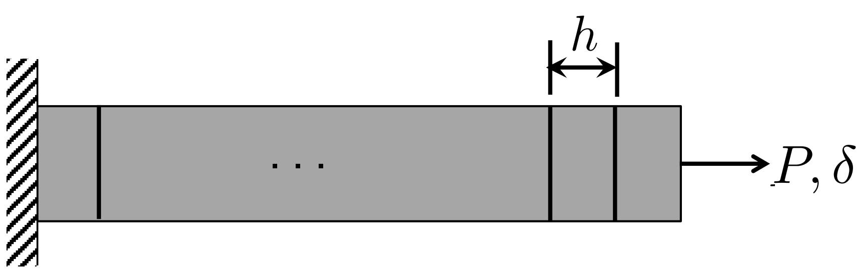
$$f(\sigma, \bar{\kappa}) = 0$$

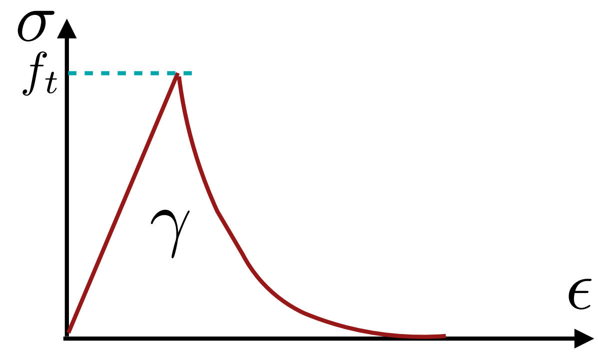


1) Integral type models (Bažant and Pijaudier-Cabot 1987)

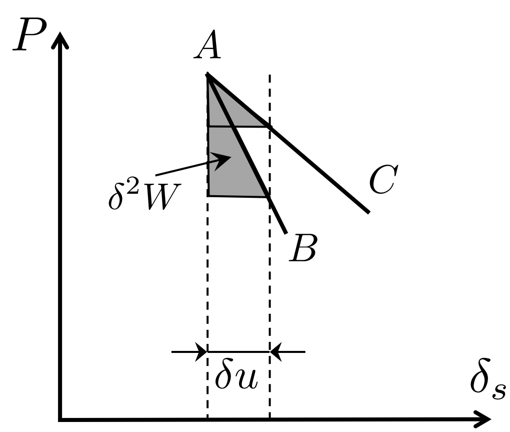
2) Gradient type models (e.g. Aifantis 1984, Bažant 1984, Mühlhaus and Aifantis 1991, Peerlings et al. 1996)

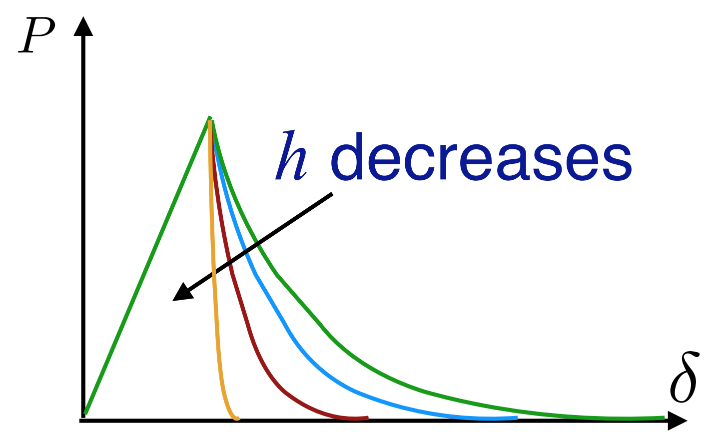
Stochastic Analysis: A Simple Example



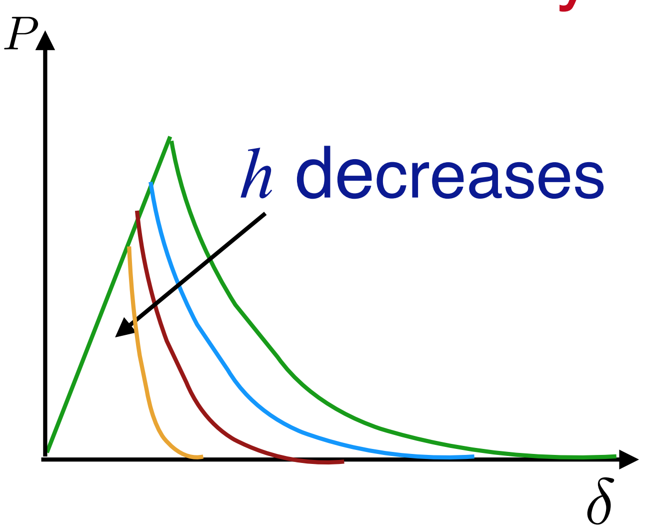


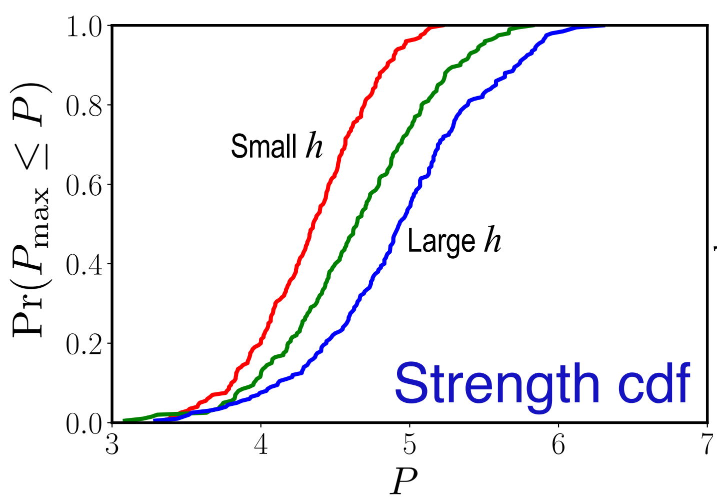
Deterministic analysis:

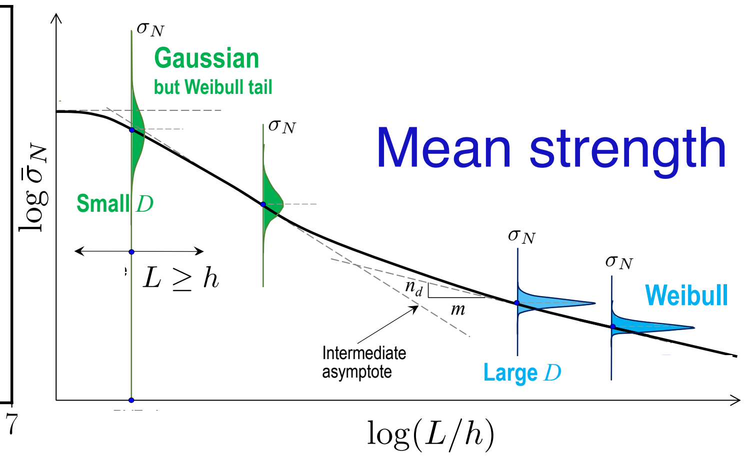




Stochastic analysis:

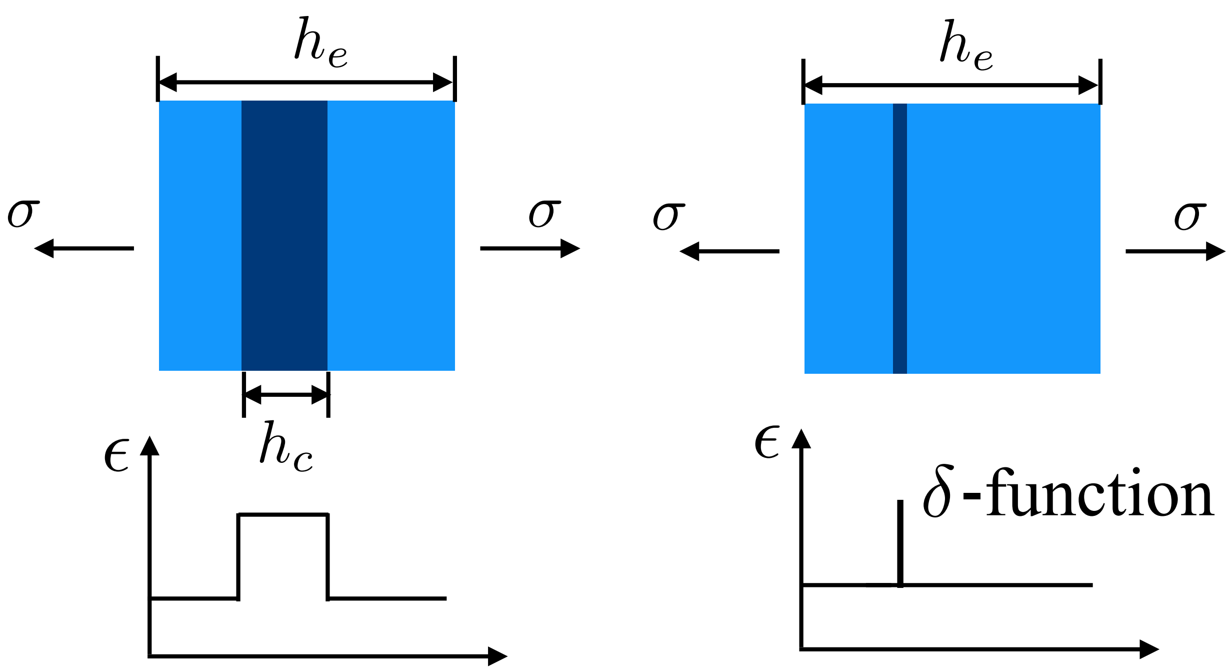


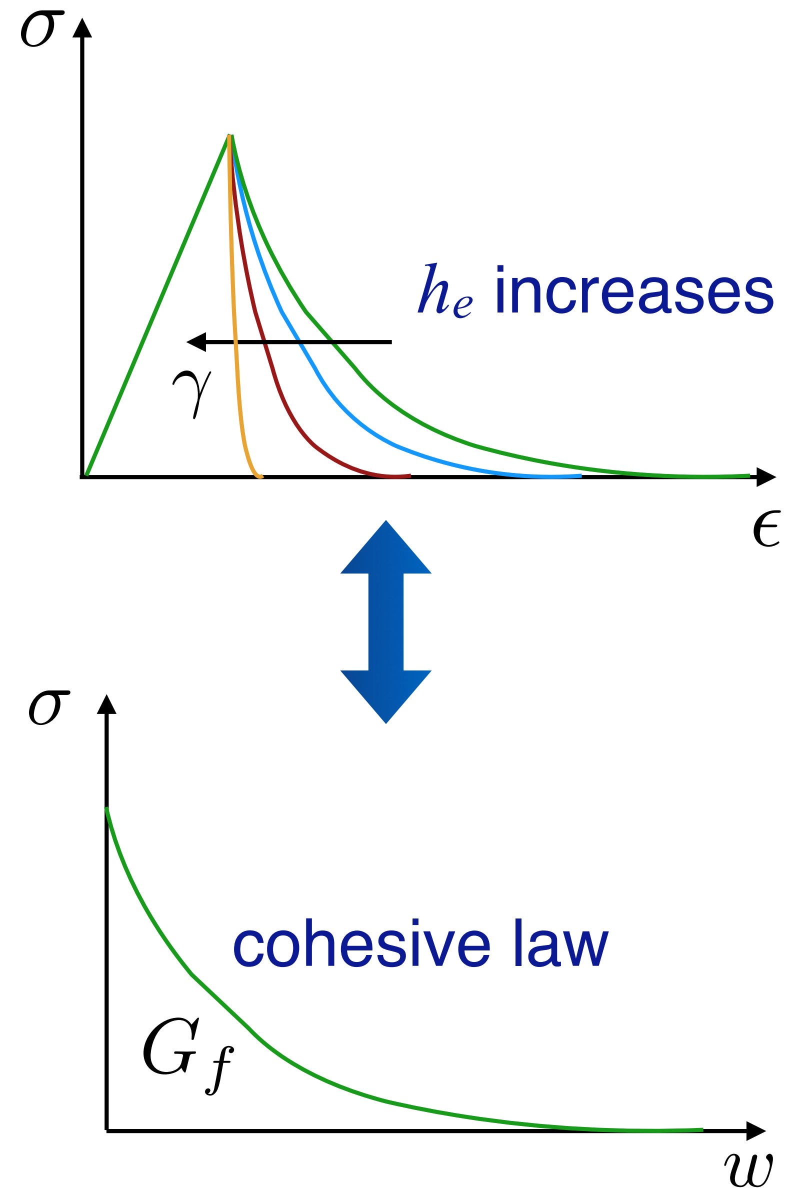




Probabilistic Crack Band Model

Regularization of fracture energy of localized crack — Conventional crack band model (Bažant and Oh 1983)



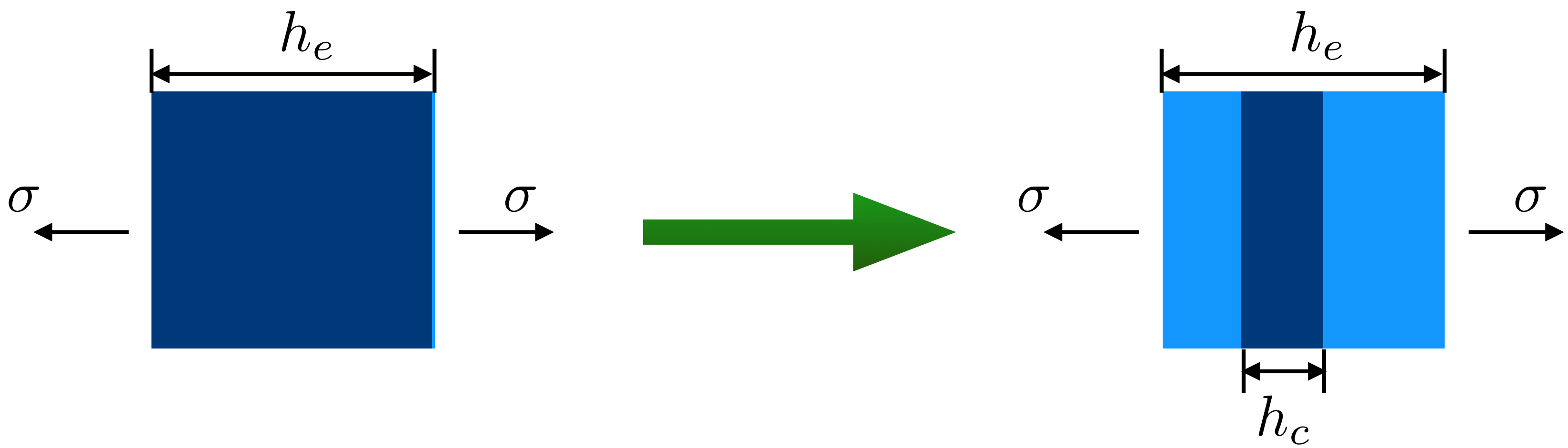


$$h_e \gamma = G_f$$

$$\gamma = \int_0^{\epsilon_u} \sigma(\epsilon) \mathrm{d}\epsilon$$

Regularization of Fracture Energy

Transition from damage initiation to damage localization:



distributed damage

localized damage

$$\Gamma = h_e \gamma_0$$

$$h_e \gamma = G_f$$

Γ is proportional to the element size, but G_f is a material constant. Therefore, it is necessary to model the transition between damage initiation and localization.