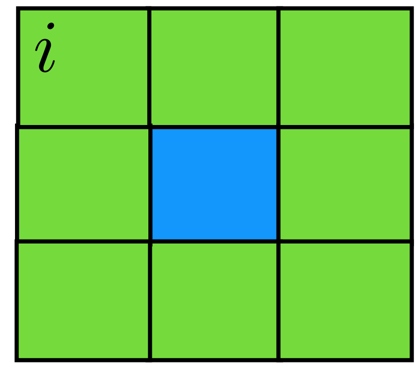
Regularization of Fracture Energy

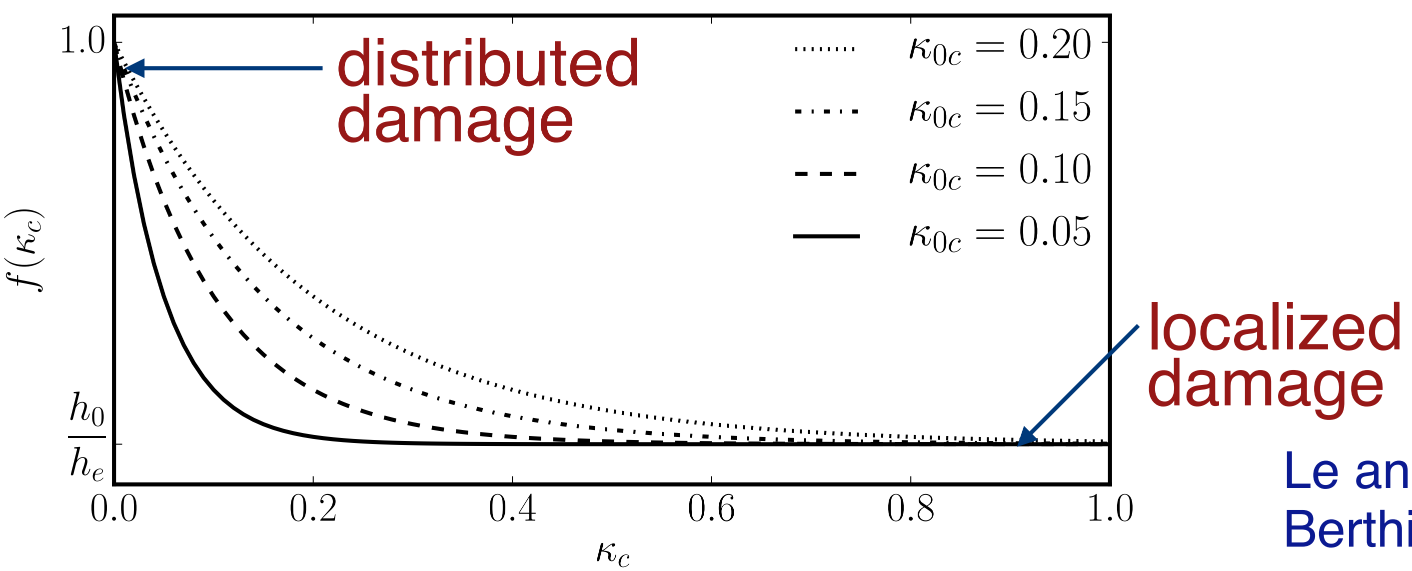
We propose a localization parameter:

$$\kappa_c = \begin{cases} \frac{1}{(n_i+n_o)} \left[\frac{(n_i+n_o+1)\cdot \max\limits_{k\le n_o+n_i} (\phi_k)}{\sum_{k=0}^{n_i+n_o} \phi_k} - 1 \right], & \text{if } \phi_0 > 0 \\ 0, & \text{if } \phi_0 = 0 \end{cases}$$



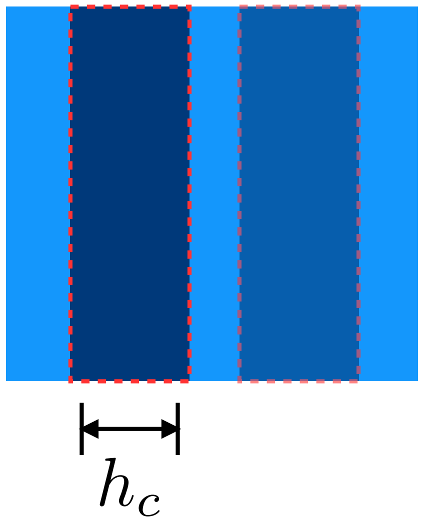
with a phenomenological transitional function for energy regularization

$$\gamma = \gamma_0 f(\kappa_c) \qquad f(\kappa_c) = \frac{h_c}{h_e} + \left(1 - \frac{h_c}{h_e}\right) \exp\left(-\frac{\kappa_c}{\kappa_{0c}}\right)$$



Le and Elias, *J. Appl. Mech.* 2016
Berthier et al. *JMPS*, 2017

Probabilistic Treatment of Random Onset of Localization Band



Damage model — incipient of localization occurs around the peak strength (Jirásek 2007).

The location of the localization band is determined by the material strength, which naturally exhibits certain spatial randomness.

Strength distribution of a material element of size h_e:

$$P(f_t) = Pr(f_t' \leq f_t) = 1 - [1 - P_1(f_t)]^{n_e}$$

$P_1(f_t) =$ strength distribution of a single localization band

$n_e =$ equivalent number of potential localization band in the element

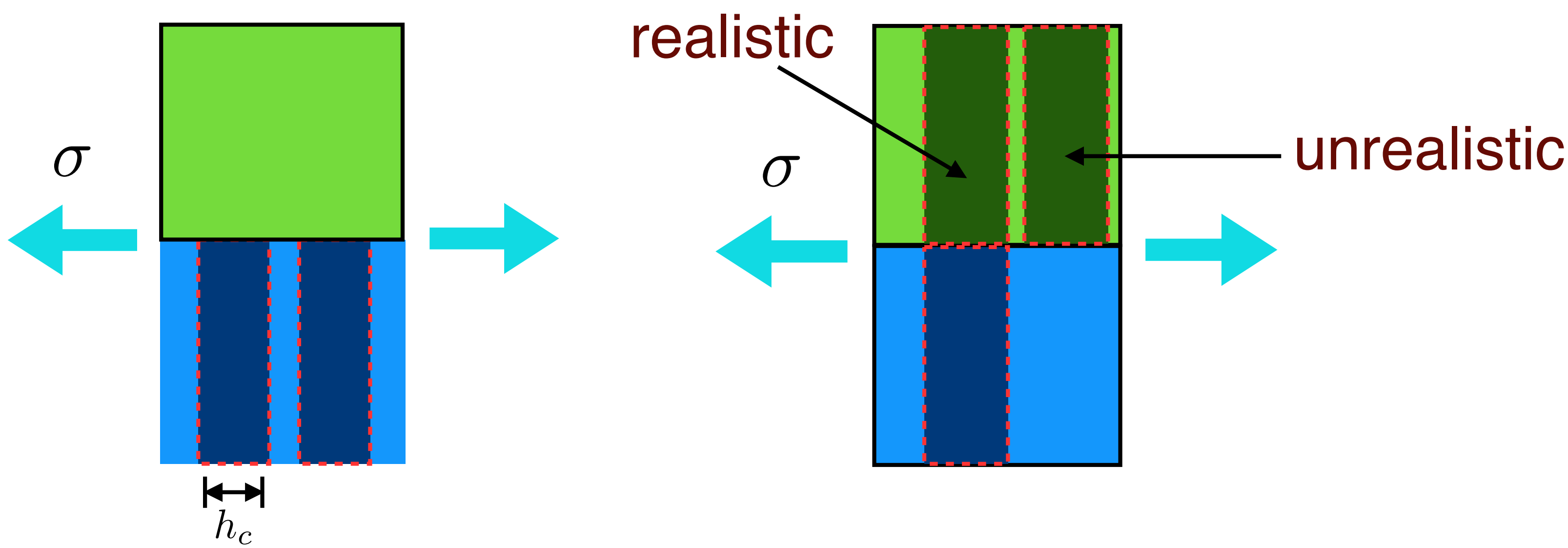
Le and Elias, *J. Appl. Mech.* 2016

Probabilistic Treatment of Random Onset of Localization Band

Strength distribution of a material element of size h_c (Bažant et al. 2009; Le et al. 2011; Bažant and Le 2017)

$$
\begin{aligned}
P_1(\sigma) &= 1 - \exp\left[-(\sigma/s_0)^m\right] && (\sigma < \sigma_{gr}) \\
P_1(\sigma) &= P_{gr} + \frac{r_f}{\sqrt{2\pi}\delta_G}\int_{\sigma_{gr}}^{\sigma} e^{-(\sigma'-\mu_G)^2/2\delta_G^2}\mathrm{d}\sigma' && (\sigma \geq \sigma_{gr})
\end{aligned}
$$

The number of potential crack bands is governed by the localization level

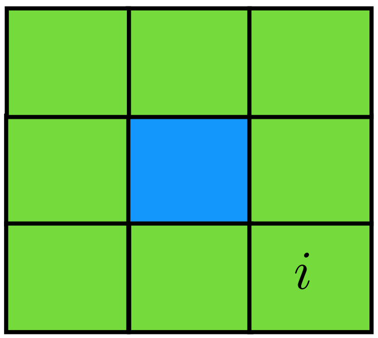


Probabilistic Treatment of Random Onset of Localization Band

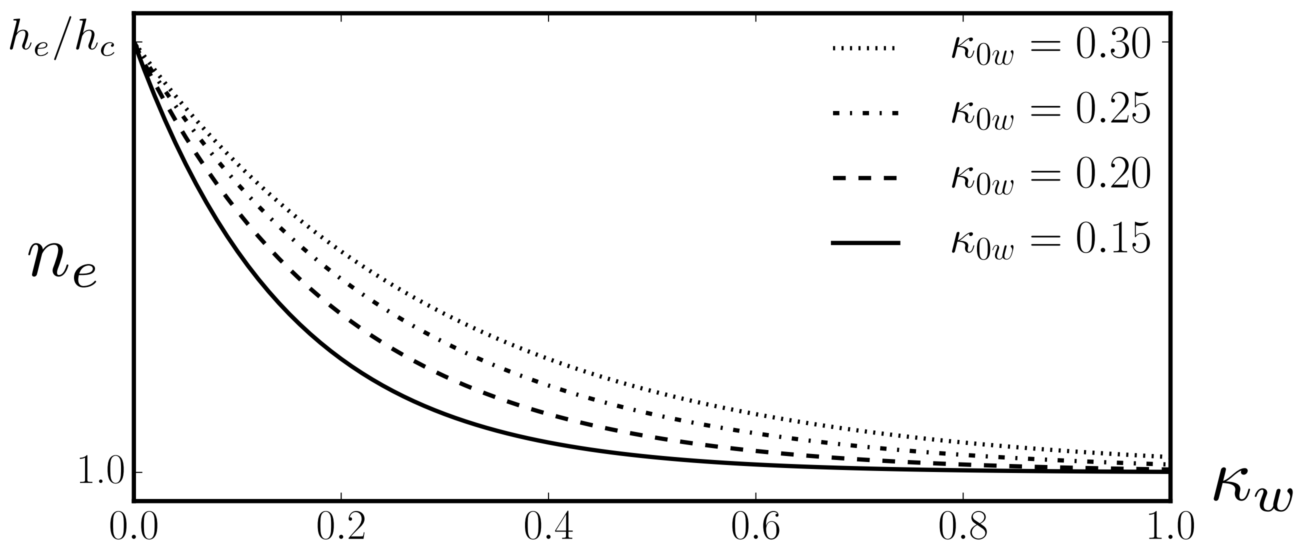
Number of potential crack bands

$$n_e = 1 + \left(\frac{h_e}{h_c} - 1\right) \exp\left(-\frac{\kappa_w}{\kappa_{0w}}\right)$$

κ_w measures the level of localization in the neighborhood



$$\kappa_w = \frac{1}{n_o - 1}\left[\frac{n_o \cdot \max\limits_{k=n_i+1}^{n_i+n_o} \phi_k}{\sum_{k=n_i+1}^{n_i+n_o} \phi_k} - 1\right]$$



Probabilistic Crack Band Model

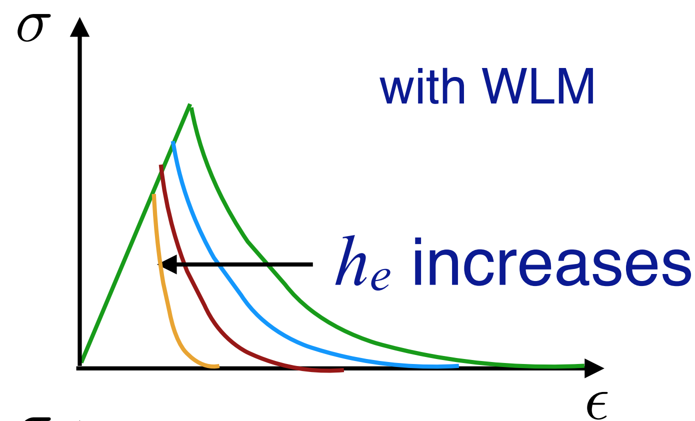
1. Regularization of fracture energy —transition from damage initiation to localization

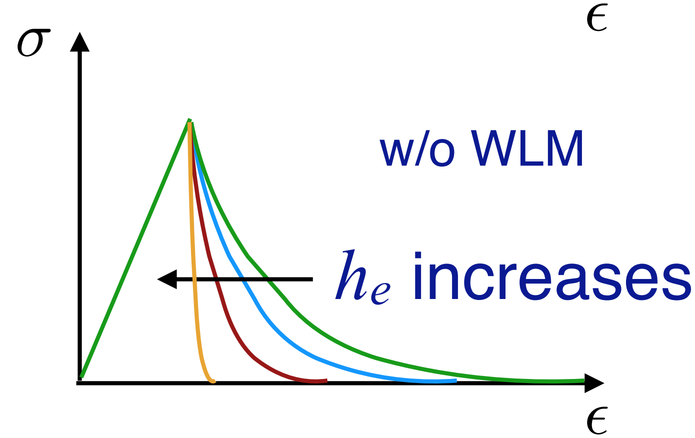
$$F_\gamma(x) = F_{G_f}[xh_c/f(\kappa_c)]$$

2. Probabilistic treatment of randomness of localization band — nonlocal information to determine the applicability of the weakest link model

$$P(f_t) = 1 - [1 - P_1(f_t)]^{n_e}$$

$$n_e = 1 + \left(\frac{h_e}{h_c} - 1\right)\exp\left(-\frac{\kappa_w}{\kappa_{0w}}\right)$$





Implications of Probabilistic Crack Band Model

Depending on the localization level, the input probability distribution of tensile strength varies with the mesh size — transitioning from a Gaussian cdf to a Weibull cdf.